63-3-2

TECH MEMO

TECHNICAL MEMORANDUM 52-61

THE RELIABILITY OF SERIES-PARALLEL MULTICHANNEL FUZE SYSTEMS

J. G. RAU
Systems Analysis Division
Fuze Department

NAVAL ORDNANCE LABORATORY CORONA

CORONA, CALIFORNIA

March 1963

NAVAL ORDNANCE LABORATORY CORONA

W. R. KURTZ, CAPT., USN
Commanding Officer

F. S. ATCHISON, Ph.D.
Technical Director

CONTENTS

ABSTRACT

This report is concerned with multichannel fuze systems in which the channels (component fuzes) are statistically identical, i.e., all have the same probability of early actuation and all have the same probability of dudding (defined as either non-function or late function). Explicit formulas are developed and numerical values are tabulated for the probability of system proper function, early function and dudding, for all possible series and/or parallel fuze configurations of n identical channels, where n = 2, 3, 4, 5. Formulas are also developed for selecting the optimum configuration so as to maximize the average probability of system proper function when the probabilities of system early function and system dudding are assigned various weights.

AUTHORIZATION

This study was undertaken under authority of WepTask RMMO-21-030/211-1/F009-08-001, Problem Assignment 1, Influence and Contact Fuze and CCM Analysis.

V. A. Brown

V. A. BROWN
Head, Systems Analysis Division

THE RELIABILITY OF SERIES-PARALLEL MULTICHANNEL FUZE SYSTEMS

INTRODUCTION

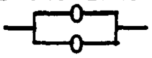
The objective of this report is to investigate the improvements in fuze reliability that may be expected by the use of multichannel fuze systems. Considerable interest has been shown in ordinary parallel and series systems (references (1)-(5)) but no complete analysis of the reliability of fuze systems consisting of both parallel and series channels has been given. As is easily seen, the complexity of such an analysis increases as the number of channels (component fuzes) increases; consequently, this report is concerned primarily with systems which contain at most five component fuzes. Other reports by the same author, which consider the more general situation, are in process of being published (references (6) and (7)).

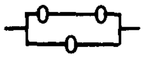
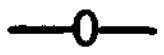
ASSUMPTIONS AND NOTATION

It is assumed in this report that the reader is familiar with the basic concepts of probability theory and the fundamentals of multichannel fuze systems. The nomenclature is basically that of reference (5). The term "fuze" refers to the target detecting device and the term "channel" refers to any of the component fuzes which individually respond to the influence of the target and jointly determine the instant of detonation. Consider for example a fuze system composed of two parallel subsystems each consisting of two component fuzes in series. Electrically there are only two channels. In a more general sense, however, there are four physical channels through which the presence of the target is detected. The term channel is used in this report in the latter sense, to refer to any of the component target detecting devices regardless of their role in the electrical circuit.

By "function" (or "actuation") of a single channel is meant the occurrence of an output signal from the channel which the fuze system combines with other channel output signals to determine whether to detonate the warhead or not, i.e., whether the system as a whole should function or not. In the systems under consideration, system function is assumed to occur only as a result of the previous function of certain individual channels or combinations of these. A channel is assumed to be capable of at most a single function which, if it occurs, is recorded in the system memory. The analysis considers only systems with infinite memory times.

With a single fuze channel we associate a probability of early function, a probability of proper function, and a probability of dudding (defined to be either non-function or late function), and the sum of these probabilities is required to be equal to one. Similarly, we associate probabilities of the same three types with the system itself. All channels are assumed to be completely independent of each other in the sense that actuation, or failure to actuate, of any channel does not affect the operation of any other channel in the system. The assumption of independence in this sense is perhaps not entirely practical since it would imply independent antennas, detonators, etc., but the complexity of the problem compels us to make this assumption.

In order to refer quickly to different multichannel fuze systems we must introduce some type of convenient notation to describe the system under consideration. If X and Y are two fuze systems, then by XpY we mean that the two systems are arranged in parallel, and by XsY we mean that the two systems are arranged in series. However, if X and Y each consist of one fuze channel, then XpY becomes 1p1, which denotes two fuze channels in parallel, and XsY becomes 1s1, which denotes two fuze channels in series. Thus 1p1 corresponds to the configuration  and 1s1 corresponds to the configuration  . In Table 1 the reader will find a complete list of fuze configurations in both diagram and notational form for the cases n = 3, 4, 5, where n is the number of channels in the particular fuze system.

Parallel systems are characterized by the property that a prescribed minimum number of channels must actuate in order to have system actuation or function. If the system consists of n channels of which at least k channels must actuate in order for the system to actuate, the system is called a (k, n) system. In order to include this minimum requirement for actuation of parallel systems we must modify the notation slightly. We make the convention that whenever the system consists of channels or subsystems in parallel, with a minimum requirement of at least k channels or subsystems actuating, we write "-k" after the indicated notation, i.e., 1p1p1p1-3 denotes four parallel channels of which at least three are required to function in order to have system function. A further example is given by (1s1)p1-1 which corresponds to the diagram  where at least one of the two subsystems  and  is required to function in order to have the entire system function. Whenever k = 1 is the only possibility for a given system, the inclusion of k in the notation for the system will be omitted. For example, (1s1)p(1s1)-1 is a valid system, but (1s1)p(1s1)-2 is effectively the same system as 1s1s1s1 and is therefore redundant; hence, for convenience, we write (1s1)p(1s1) instead of (1s1)p(1s1)-1. Furthermore, in all systems of five fuzes we assume k = 1 unless otherwise specified.

RELIABILITY OF SERIES-PARALLEL SYSTEMS

We make the following conventions in terminology:

P = probability of proper function of the system.

E = probability of early function of the system.

D = probability of dudding of the system.

p = probability of proper function of a single channel.

e = probability of early function of a single channel.

d = probability of dudding of a single channel.

For the remainder of this report we assume that the individual channels in any given system are identical in the probability sense, i.e., all p's are equal, all e's are equal and all d's are equal. For a system with n such channels arranged in series we have

$$P = \sum_{j=0}^{n-1} \binom{n}{j} p^{n-j} e^{j} = (1-d)^{n} - e^{n} \tag{1}$$

$$E = e^{n} \tag{2}$$

$$D = 1 - (1-d)^{n} \tag{3}$$

For a (k, n) system with identical channels we have

$$P = 1 - \sum_{j=k}^{n} \binom{n}{j} e^{j}(1-e)^{n-j} - \sum_{j=n-k+1}^{n} \binom{n}{j} d^{j} (1-d)^{n-j} \tag{4}$$

$$E = \sum_{j=k}^{n} \binom{n}{j} e^{j} (1-e)^{n-j} \tag{5}$$

$$D = \sum_{j=n-k+1}^{n} \binom{n}{j} d^{j} (1-d)^{n-j} \tag{6}$$

In Tables 2 to 5 these probability formulas are given explicitly for every possible series and/or parallel fuze configuration of n identical channels, for the cases n = 2, 3, 4, 5. In Tables 6 to 17 will be found numerical values of P, E, and D for given values of e and d, for identical-channel fuzes. No tables are given in this report for systems involving non-identical channels; however tables have been developed through the use of an IBM 7070 computer for configurations involving two non-identical channels. The reliability of dissimilar-channel fuzes may be calculated by recursion formulas given in reference (6).

OPTIMUM CONFIGURATIONS

In comparing various multichannel fuze systems it is sometimes desirable to select a system in which the probability of early function is of prime importance and the probability of dudding of the system is secondary, or vice-versa. Thus, a logical method of selection would be to assign weighting factors to each of these probabilities and to determine a weighted probability of proper function for given values of E and D (or for given values of e and d if the channels are identical). In this manner we are able to make a reasonable comparison between multichannel fuze systems and, consequently, are able to select a fuze system that is optimum with respect to these weighted probabilities.

Considering now only multichannel fuze systems in which the channels are identical in the probability sense, i.e., having the same e's and d's, let us define the following quantities:

W_E = weighting for the probability of early function, and

W_D = weighting for the probability of dudding, where

$$W_E + W_D = 2. \tag{7}$$

Thus, for the weighted probability P_W of system proper function we define

$$P_W = 1 - W_E \cdot E - W_D \cdot D. \tag{8}$$

From equations (7) and (8) we also obtain

$$P_W = 1 - W_E \cdot E - (2 - W_E)D = 1 - W_E\,(E - D) - 2D \text{ and}$$

$$P_W = 1 - (2 - W_D)E - W_D \cdot D = 1 - W_D(D - E) - 2E.$$

If we weight E and D equally, i.e., if $W_E = W_D = 1$, then we obtain $P_W = 1 - E - D$, which we recognize as the probability of proper function in general.

The expressions given above for P_W refer to a particular pair of values of e and d. If it is desirable to compare multichannel fuze systems for not only one (e, d) pair but for all possible (e, d) pairs as e and d range through a given finite set of values, then we define $\bar{P}_W$, the average system proper function probability over the intervals $0 \leq e \leq a \leq 1$ and $0 \leq d \leq b \leq 1$, by the equation

$$\bar{P}_W = \frac{\sum\limits_{\substack{0 \leq e \leq a \\ 0 \leq d \leq b}} P_W}{N} = N - \frac{\sum\limits_{\substack{0 \leq e \leq a \\ 0 \leq d \leq b}} (W_E \cdot E + W_D \cdot D)}{N}$$

where we sum over a given finite number N of (e, d) pairs for which $0 \leq e \leq a$ and $0 \leq d \leq b$.

In Tables 18 to 22, numerical values of $\bar{P}_W$ are given for multichannel fuze systems of n channels for the cases n = 2, 3, 4, 5 over the five (e, d) intervals:

$$\begin{Bmatrix} 0 \leq e \leq 0.1 \\ 0 \leq d \leq 0.1 \end{Bmatrix}, \begin{Bmatrix} 0 \leq e \leq 0.15 \\ 0 \leq d \leq 0.15 \end{Bmatrix}, \begin{Bmatrix} 0 \leq e \leq 0.2 \\ 0 \leq d \leq 0.2 \end{Bmatrix}, \begin{Bmatrix} 0 \leq e \leq 0.1 \\ 0 \leq d \leq 0.2 \end{Bmatrix}, \begin{Bmatrix} 0 \leq e \leq 0.2 \\ 0 \leq d \leq 0.1 \end{Bmatrix}.$$

The probabilities e and d are incremented in steps of 0.05. The weighting patterns are defined by setting W_E = 1.5, 1.33, 1.0, 0.67 and 0.5 in equation (7). The results are discussed in the conclusions of this report.

In reference (5) it was shown that for a fuze consisting of n channels in parallel, of which at least k channels must function in order to have fuze function, the maximum reliability, i.e., probability of fuze proper function, is obtained when we select k according to the condition

$$k = \begin{cases} \dfrac{n+1}{2} \text{ for n odd} \\[2ex] \dfrac{n}{2} \text{ or } \dfrac{n}{2} + 1 \text{ for n even.} \end{cases}$$

From equations (5) and (6) we have

$$E(x) = \sum_{j=k}^{n} \binom{n}{j} x^j (1 - x)^{n-j} \tag{9}$$

and

$$D(x) = \sum_{j=n-k+1}^{n} \binom{n}{j} x^j (1-x)^{n-j} . \tag{10}$$

Hence, when n is odd and we select k such that $k = \frac{n+1}{2}$, we observe that $E(x) = D(x)$, where, in particular, $E(e) = E$ and $D(d) = D$. Furthermore, for n odd and $k = \frac{n+1}{2}$, $\bar{P}_W$ is independent of W_E and W_D when e and d range over the same intervals. Finally, suppose n is even. If $k = \frac{n}{2}$, let $P_{\frac{n}{2}}$, $E_{\frac{n}{2}}$, $D_{\frac{n}{2}}$ be the probabilities associated with the system for which $k = \frac{n}{2}$; if $k = \frac{n}{2}+1$, let $P_{\frac{n}{2}+1}$, $E_{\frac{n}{2}+1}$, $D_{\frac{n}{2}+1}$ be the probabilities associated with the system for which $k = \frac{n}{2}+1$. From equations (5) and (6) we have

$$E_{\frac{n}{2}} = \sum_{j=\frac{n}{2}}^{n} \binom{n}{j} e^j (1-e)^{n-j} \tag{11}$$

$$D_{\frac{n}{2}} = \sum_{j=\frac{n}{2}+1}^{n} \binom{n}{j} d^j (1-d)^{n-j} \tag{12}$$

$$E_{\frac{n}{2}+1} = \sum_{j=\frac{n}{2}+1}^{n} \binom{n}{j} e^j (1-e)^{n-j} \tag{13}$$

$$D_{\frac{n}{2}+1} = \sum_{j=\frac{n}{2}}^{n} \binom{n}{j} d^j (1-d)^{n-j} \tag{14}$$

Hence, from equations (11) to (14) we see that

$$E_{\frac{n}{2}} = \binom{n}{\frac{n}{2}} e^{n/2} (1-e)^{n/2} + E_{\frac{n}{2}+1} \tag{15}$$

$$D_{\frac{n}{2}} = D_{\frac{n}{2}+1} - \binom{n}{\frac{n}{2}} d^{n/2} (1-d)^{n/2}; \tag{16}$$

consequently,

$$\text{when } W_E > W_D, \qquad \bar{P}_{\frac{n}{2}+1} > \bar{P}_{\frac{n}{2}} ;$$

$$\text{when } W_E = W_D, \qquad \bar{P}_{\frac{n}{2}+1} = \bar{P}_{\frac{n}{2}} ;$$

$$\text{and when } W_E < W_D, \qquad \bar{P}_{\frac{n}{2}+1} < \bar{P}_{\frac{n}{2}} ,$$

provided e and d range over the same range of values. Therefore, when system early function is more important (in the sense of a heavier weighting) than dudding, the choice $k = \frac{n}{2} + 1$ provides a higher average system proper function probability; similarly, when system dudding is more important than system early function, the choice $k = \frac{n}{2}$ provides a higher average system proper function probability.

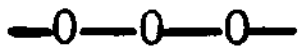
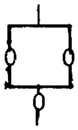
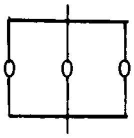
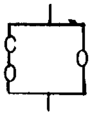
CONCLUSIONS

It is obvious from the formulas associated with such systems, as it is intuitively, that series systems minimize early function and parallel systems minimize dudding; however, the results of Tables 18 to 22 show that no parallel and series combination system does both. Furthermore, we have shown that for systems consisting of two, three, four, and five channels, certain ordinary parallel systems are more desirable with respect to reliability than any parallel and series combination of the same number of component fuzes. Using the criteria of the preceding section, we have listed in Table 22 the optimum n-channel system(s) for the cases n = 2, 3, 4, 5. From this table we may conclude that a system consisting of five identical channels in parallel, of which at least three must function in order to have entire system function, is optimum with respect to reliability.

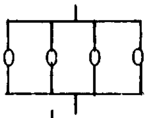
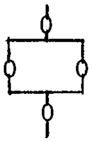
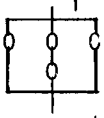
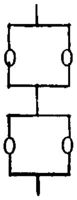
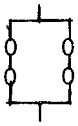
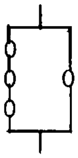
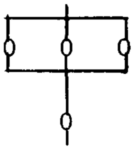
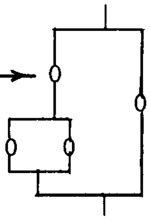
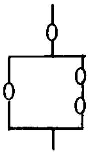
It has been observed in reference (5) that in a parallel system of n identical channels in which the probability of system early function equals the probability of system dudding, the maximum reliability increases as n increases from even to odd but remains constant as n increases from odd to even. As a consequence of our earlier discussion with regard to weightings, the preceding statement is also independent of the weightings for the early function and dudding probabilities. Furthermore, if E, the probability of system early function and D, the probability of system dudding, are weighted equally, then, as a consequence of the above results from reference (5), the average system proper function probability increases as n increases from even to odd but remains constant as n increases from odd to even, provided e (the probability of individual fuze channel early function) and d (the probability of individual channel dudding) range over the same finite set of values. This is not necessarily true, however, when E and D are assigned unequal weights, as may be seen from Table 23 and Fig. 1 to 5, which present the average probability of proper function, $\bar{P}_W$, for a (k, n) system of identical parallel channels, for values of n from 3 to 9 and selected values of k.

Table 1. Notation and Schematic Diagrams

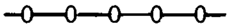
<u>n = 3</u>

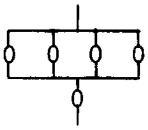
1s1s1 ↔		(1p1)s1 ↔	
1p1p1 ↔		(1s1)p1 ↔	

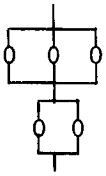
<u>n = 4</u>

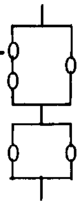
1s1s1s1 ↔		1p1p1p1 ↔	
1s(1p1)s1 ↔		1p(1s1)p1 ↔	
(1p1)s(1p1) ↔		(1s1)p(1s1) ↔	
(1s1s1)p1 ↔		(1p1p1)s1 ↔	
[1s(1p1)] p1 ↔		[1p(1s1)] s1 ↔	

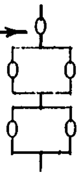
n = 5

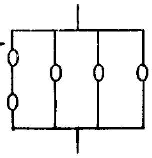
lslslslsl ↔ 

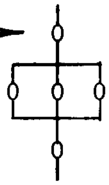
(lplplpl)sl ↔ 

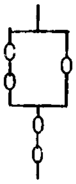
(lplpl)s(lpl) ↔ 

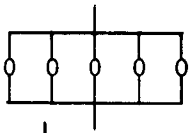
[(lsl)pl] s(lpl) ↔ 

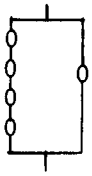
ls(lpl)s(lpl) ↔ 

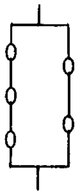
(lsl)plplpl ↔ 

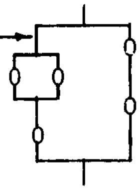
ls(lplpl)sl ↔ 

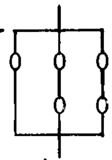
[(lsl)pl] slsl ↔ 

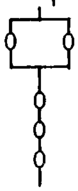
lplplplpl ↔ 

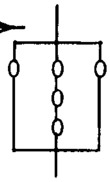
(lslslsl)pl ↔ 

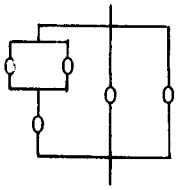
(lslsl)p(lsl) ↔ 

[(lpl)sl] p(lsl) ↔ 

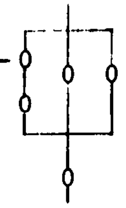
lp(lsl)p(lsl) ↔ 

(lpl)slslsl ↔ 

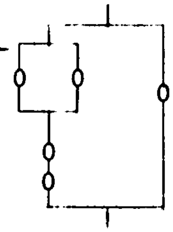
lp(lslsl)pl ↔ 

[(lpl)sl] plpl ↔ 

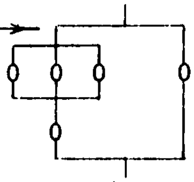
[(lsl)plpl] sl



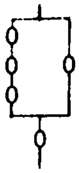
[(lpl)slsl] pl



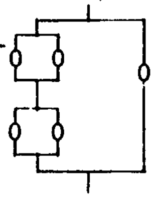
[(lplpl)sl] pl



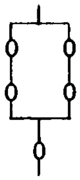
[(lslsl)pl] sl



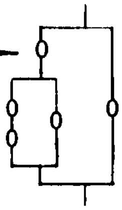
[(lpl)s(lpl)] pl



[(lsl)p(lsl)] sl



{ls [(lsl)pl]} pl



{lp [(lpl)sl]} sl

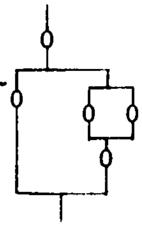


Table 2

n = 2

	Probability (P) of Proper Function	Probability (E) of Early Function	Probability (D) of Dudding
1s1	$P=(1-d)^2-e^2$	$E=e^2$	$D=1-(1-d)^2$
1p1-1	$P=(1-e)^2-d^2$	$E=1-(1-e)^2$	$D=d^2$

Table 3

n = 3

1s1s1	$P=(1-d)^3-e^3$	$E=e^3$	$D=1-(1-d)^3$
1p1p1-1	$P=(1-e)^3-d^3$	$E=1-(1-e)^3$	$D=d^3$
1p1p1-2	$P=1-3e^2+2e^3-3d^2+2d^3$	$E=3e^2-2e^3$	$D=3d^2-2d^3$
(1p1)s1	$P=1-2e^2+e^3-d-d^2+d^3$	$E=2e^2-e^3$	$D=d+d^2-d^3$
(1s1)p1	$P=1-e-e^2+e^3-d^2+d^3$	$E=e+e^2-e^3$	$D=2d^2-d^3$

Table 4

n = 4

1s1s1s1	$P=(1-d)^4-e^4$	$E=e^4$	$D=1-(1-d)^4$
1p1p1p1-1	$P=(1-e)^4-d^4$	$E=1-(1-e)^4$	$D=d^4$
1p1p1p1-2	$P=1-6e^2+8e^3-3e^4-4d^3+3d^4$	$E=6e^2+3e^4-8e^3$	$D=4d^3-3d^4$
1p1p1p1-3	$P=1-4e^3+3e^4-3d^4+8d^3-6d^2$	$E=4e^3-3e^4$	$D=6d^2-8d^3+3d^4$
(1p1)s(1p1)	$P=1-4e^2+4e^3-e^4-2d^2+d^4$	$E=4e^2-4e^3+e^4$	$D=2d^2-d^4$
(1s1)p(1s1)	$P=1-2e^2+e^4-4d^2+4d^3-d^4$	$E=2e^2-e^4$	$D=4d^2-4d^3+d^4$
(1s1s1)p1	$P=1-e-e^3+e^4-3d^2+3d^3-d^4$	$E=e+e^3-e^4$	$D=3d^2-3d^3+d^4$
(1p1p1)s1-1	$P=1-3e^2+3e^3-e^4-d-d^3+d^4$	$E=3e^2-3e^3+e^4$	$D=d+d^3-d^4$
(1p1p1)s1-2	$P=1-3e^3+2e^4-d-3d^2+5d^3-2d^4$	$E=3e^3-2e^4$	$D=d+3d^2-5d^3+2d^4$
(1p1)s1s1	$P=1-2e^3+e^4-2d+2d^3-d^4$	$E=2e^3-e^4$	$D=2d-2d^3+d^4$
(1s1)p1p1-1	$P=1-2e+2e^3-e^4-2d^3+d^4$	$E=2e-2e^3+e^4$	$D=2d^3-d^4$
(1s1)p1p1-2	$P=1-e^2-2e^3+2e^4-5d^2+6d^3-2d^4$	$E=e^2+2e^3-2e^4$	$D=5d^2-6d^3+2d^4$
1p [1s(1p1)]	$P=1-e-2e^2+3e^3-e^4-d^2-d^3+d^4$	$E=e+2e^2-3e^3+e^4$	$D=d^2+d^3-d^4$
1s [1p(1s1)]	$P=1-e^2-e^3+e^4-d-2d^2+3d^3-d^4$	$E=e^2+e^3-e^4$	$D=d+2d^2-3d^3+d^4$

Table 5 $n = 5$	Probability (P) of Proper Function	Probability (E) of Early Function	Probability (D) of Dudding
1s1s1s1s1	$P=(1-d)^5-e^5$	$E=e^5$	$D=1-(1-d)^5$
1p1p1p1p1-1	$P=(1-e)^5-d^5$	$E=1-(1-e)^5$	$D=d^5$
1p1p1p1p1-2	$P=1-10e^2+20e^3-15e^4+4e^5-5d^4+4d^5$	$E=10e^2-20e^3+15e^4-4e^5$	$D=5d^4-4d^5$
1p1p1p1p1-3	$P=1-10e^3+15e^4-6e^5-10d^3+15d^4-6d^5$	$E=10e^3-15e^4+6e^5$	$D=10d^3-15d^4+6d^5$
1p1p1p1p1-4	$P=1-5e^4+4e^5-10d^2+20d^3-15d^4+4d^5$	$E=5e^4-4e^5$	$D=10d^2-20d^3+15d^4-4d^5$
1s(1p1p1p1)	$P=1-4e^2+6e^3-4e^4+e^5-d-d^4+d^5$	$E=4e^2-6e^3+4e^4-e^5$	$D=d+d^4-d^5$
1p(1s1s1s1)	$P=1-e-e^4+e^5-4d^2+6d^3-4d^4+d^5$	$E=e+e^4-e^5$	$D=4d^2-6d^3+4d^4-d^5$
(1p1p1)s(1p1)	$P=1-6e^2+9e^3-5e^4+e^5-d^2-d^3+d^5$	$E=6e^2-9e^3+5e^4-e^5$	$D=d^2-d^3-d^5$
(1s1s1)p(1s1)	$P=1-e^2-e^3+e^5-6d^2+9d^3-5d^4+d^5$	$E=e^2+e^3-e^5$	$D=6d^2-9d^3+5d^4-d^5$
[(1s1)p1] s(1p1)	$P=1-2e^2-e^3+3e^4-e^5-3d^2+d^3+2d^4-d^5$	$E=2e^2+e^3-3e^4+e^5$	$D=3d^2-d^3-2d^4-d^5$
[(1p1)s1] p(1s1)	$P=1-3e^2+e^3+2e^4-e^5-2d^2-d^3+3d^4-d^5$	$E=3e^2-e^3-2e^4+e^5$	$D=2d^2+d^3-3d^4+d^5$
1s(1p1)s(1p1)	$P=1-4e^3+4e^4-e^5-d-2d^2+2d^3+d^4-d^5$	$E=4e^3-4e^4+e^5$	$D=d+2d^2-2d^3-d^4+d^5$
1p(1s1)p(1s1)	$P=1-e-2e^2+2e^3+e^4-e^5-4d^3+4d^4-d^5$	$E=e+2e^2-2e^3-e^4+e^5$	$D=4d^3-4d^4+d^5$
(1s1)p1p1p1	$P=1-3e+2e^2+2e^3-3e^4+e^5-2d^4+d^5$	$E=3e-2e^2-2e^3+3e^4-e^5$	$D=2d^4-d^5$
(1p1)s1s1s1	$P=1-2e^4+e^5-3d+2d^2+2d^3-3d^4+d^5$	$E=2e^4-e^5$	$D=3d-2d^2-2d^3+3d^4-d^5$
(1p1p1)s1s1	$P=1-3e^3+3e^4-e^5-2d+d^2-d^3+2d^4-d^5$	$E=3e^3-3e^4+e^5$	$D=2d-d^2+d^3-2d^4+d^5$
(1s1s1)p1p1	$P=1-2e+e^2-e^3+2e^4-e^5-3d^3+3d^4-d^5$	$E=2e-e^2+e^3-2e^4+e^5$	$D=3d^3-3d^4-d^5$
[(1s1)p1]s1s1	$P=1-e^3-e^4+e^5-2d-d^2+5d^3-4d^4+d^5$	$E=e^3+e^4-e^5$	$D=2d+d^2-5d^3+4d^4-d^5$
[(1p1)s1]p1p1	$P=1-2e-e^2+5e^3-4e^4+e^5-d^3-d^4+d^5$	$E=2e+e^2-5e^3+4e^4-e^5$	$D=d^3-d^4-d^5$
[(1s1)p1p1]s1	$P=1-2e^2+2e^4-e^5-d-2d^3+3d^4-d^5$	$E=2e^2-2e^4+e^5$	$D=d+2d^3-3d^4-d^5$
[(1p1)s1s1]p1	$P=1-e-2e^3+3e^4-e^5-2d^2+2d^4-d^5$	$E=e+2e^3-3e^4+e^5$	$D=2d^2-2d^4-d^5$
[(1p1p1)s1] p1	$P=1-e-3e^2+6e^3-4e^4+e^5-d^2-d^4+d^5$	$E=e+3e^2-6e^3+4e^4-e^5$	$D=d^2+d^4-d^5$
[(1s1s1)p1] s1	$P=1-e^2-e^4+e^5-d-3d^2+6d^3-4d^4+d^5$	$E=e^2+e^4-e^5$	$D=d+3d^2-6d^3+4d^4-d^5$
[(1p1)s(1p1)]p1	$P=1-e-4e^2+8e^3-5e^4+e^5-2d^3+d^5$	$E=e-4e^2-8e^3+5e^4-e^5$	$D=2d^3-d^5$
[(1s1)p(1s1)] s1	$P=1-2e^3+e^5-d-4d^2+8d^3-5d^4+d^5$	$E=2e^3-e^5$	$D=d+4d^2-8d^3+5d^4-d^5$
{1s[(1s1)p1]}p1	$P=1-e-e^2+2e^4-e^5+d^2-2d^3+3d^4-d^5$	$E=e+e^2-2e^4+e^5$	$D=d^2+2d^3-3d^4-d^5$
{1p[(1p1)s1]}s1	$P=1+e^2-2e^3+3e^4-e^5-d-d^2+2d^4-d^5$	$E=e^2+2e^3-3e^4+e^5$	$D=d-d^2-2d^4+d^5$

Table 6. Probability of Proper Function

n = 2

e	d	lpl-l	lsl
0	0	1.000	1.000
	.05	.997	.902
	.1	.990	.810
	.15	.977	.722
	.2	.960	.640
	.25	.937	.562
	.3	.910	.490
.05	0	.902	.997
	.05	.899	.899
	.1	.892	.807
	.15	.879	.719
	.2	.862	.637
	.25	.839	.559
	.3	.812	.487
.1	0	.810	.990
	.05	.807	.892
	.1	.800	.800
	.15	.787	.712
	.2	.770	.630
	.25	.747	.552
	.3	.720	.480
.15	0	.722	.977
	.05	.719	.879
	.1	.712	.787
	.15	.699	.699
	.2	.682	.617
	.25	.659	.539
	.3	.632	.467
.2	0	.640	.960
	.05	.637	.862
	.1	.630	.770
	.15	.617	.682
	.2	.600	.600
	.25	.577	.522
	.3	.550	.450
.25	0	.562	.937
	.05	.559	.839
	.1	.552	.747
	.15	.539	.659
	.2	.522	.577
	.25	.499	.499
	.3	.472	.427
.3	0	.490	.910
	.05	.487	.812
	.1	.480	.720
	.15	.467	.632
	.2	.450	.550
	.25	.427	.472
	.3	.400	.400

Table 7. Probability of Early Function

n = 2

e	1p1-1	1s1
0	.000	.000
.05	.098	.003
.1	.190	.010
.15	.278	.023
.2	.360	.040
.25	.438	.063
.3	.510	.090

Table 8. Probability of Dudding

n = 2

d	1p1-1	1s1
0	.000	.000
.05	.003	.098
.1	.010	.190
.15	.023	.278
.2	.040	.360
.25	.063	.438
.3	.090	.510

Table 9. Probability of Proper Function

n = 3

e	d	lslsl	(lplpl)-1	(lplpl)-2	(lpl)sl	(lsl)pl
0	0	1.000	1.000	1.000	1.000	1.000
	.05	.857	1.000	.993	.948	.995
	.1	.729	.999	.972	.891	.981
	.15	.614	.997	.939	.831	.958
	.2	.512	.992	.896	.768	.928
	.25	.422	.984	.844	.703	.891
	.3	.343	.973	.784	.637	.847
.05	0	1.000	.857	.993	.995	.948
	.05	.857	.857	.986	.943	.943
	.1	.729	.856	.965	.886	.929
	.15	.614	.854	.932	.826	.906
	.2	.512	.849	.889	.763	.876
	.25	.422	.842	.837	.698	.838
	.3	.343	.830	.777	.632	.795
.1	0	.999	.729	.972	.981	.891
	.05	.856	.729	.965	.929	.886
	.1	.728	.728	.944	.872	.872
	.15	.613	.726	.911	.812	.849
	.2	.511	.721	.868	.749	.819
	.25	.421	.713	.816	.684	.782
	.3	.342	.702	.756	.618	.738
.15	0	.997	.614	.939	.958	.831
	.05	.854	.614	.932	.906	.826
	.1	.726	.613	.911	.849	.812
	.15	.611	.611	.879	.789	.789
	.2	.509	.606	.835	.726	.759
	.25	.419	.599	.783	.662	.722
	.3	.340	.587	.723	.595	.678
.2	0	.992	.512	.896	.928	.768
	.05	.849	.512	.889	.876	.763
	.1	.721	.511	.868	.819	.749
	.15	.606	.509	.835	.759	.726
	.2	.504	.504	.792	.696	.696
	.25	.414	.496	.740	.631	.659
	.3	.335	.485	.680	.565	.615
.25	0	.984	.422	.844	.891	.703
	.05	.842	.422	.837	.838	.698
	.1	.713	.421	.816	.782	.684
	.15	.599	.419	.783	.722	.662
	.2	.496	.414	.740	.659	.631
	.25	.406	.406	.688	.594	.594
	.3	.327	.395	.628	.528	.550
.3	0	.973	.343	.784	.847	.637
	.05	.830	.343	.777	.795	.632
	.1	.702	.342	.756	.738	.618
	.15	.587	.340	.723	.678	.595
	.2	.485	.335	.680	.615	.565
	.25	.395	.327	.628	.550	.528
	.3	.316	.316	.568	.484	.484

Table 10. Probability of Early Function

n = 3

e	lslsl	(lplpl)-1	(lplpl)-2	(lpl)sl	(lsl)pl
0	.000	.000	.000	.000	.000
.05	.000	.143	.007	.005	.052
.1	.001	.271	.028	.019	.109
.15	.003	.386	.061	.042	.169
.2	.008	.488	.104	.072	.232
.25	.016	.578	.156	.109	.297
.3	.027	.657	.216	.153	.363

Table 11. Probability of Dudding

n = 3

d	lslsl	(lplpl)-1	(lplpl)-2	(lpl)sl	(lsl)pl
0	.000	.000	.000	.000	.000
.05	.143	.000	.007	.052	.005
.1	.271	.001	.028	.109	.019
.15	.386	.003	.061	.169	.042
.2	.488	.008	.104	.232	.072
.25	.578	.016	.156	.297	.109
.3	.657	.027	.216	.363	.153

Table 12. Probability of Proper Function n = 4

e	d	1s1s1s1	1p1p1p1-1	1p1p1p1-2	1p1p1p1-3	(1p1)s(1p1)	(1s1)p(1s1)	(1s1s1)p1	(1p1p1)s1-1	(1p1p1)s1-2	(1p1)s1s1	(1s1)p1p1-1	(1s1)p1p1-2	1p[1s(1p1)]	1s[1p(1s1)]
0	0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	.05	.815	1.000	1.000	.986	.995	.990	.993	.950	.943	.900	1.000	.988	.997	.945
	.1	.656	1.000	.996	.948	.980	.964	.973	.899	.875	.802	.998	.956	.989	.883
	.15	.522	.999	.988	.890	.956	.923	.942	.847	.798	.706	.994	.907	.975	.815
	.2	.410	.998	.973	.819	.922	.870	.902	.794	.717	.614	.986	.845	.954	.742
	.25	.316	.996	.949	.738	.879	.809	.855	.738	.633	.527	.973	.773	.926	.668
	.3	.240	.992	.916	.652	.828	.740	.803	.681	.549	.446	.954	.696	.891	.593
.05	0	1.000	.815	.986	1.000	.990	.995	.950	.993	1.000	1.000	.900	.997	.945	.997
	.05	.815	.815	.986	.986	.986	.986	.943	.943	.943	.900	.900	.986	.942	.943
	.1	.656	.814	.982	.947	.971	.959	.923	.892	.874	.802	.898	.953	.934	.880
	.15	.522	.814	.974	.890	.946	.918	.892	.840	.798	.706	.894	.904	.920	.812
	.2	.410	.813	.959	.819	.912	.865	.852	.786	.716	.614	.886	.842	.899	.740
	.25	.316	.811	.935	.738	.869	.804	.805	.731	.632	.527	.873	.771	.871	.665
	.3	.240	.806	.902	.651	.819	.735	.753	.674	.548	.446	.854	.693	.836	.590
.1	0	1.000	.656	.948	.996	.964	.980	.899	.973	.997	.998	.802	.988	.883	.989
	.05	.815	.656	.947	.982	.959	.971	.892	.923	.940	.898	.802	.976	.880	.934
	.1	.656	.656	.944	.944	.944	.944	.872	.872	.872	.800	.800	.944	.872	.872
	.15	.522	.656	.936	.887	.919	.903	.841	.820	.796	.704	.796	.895	.857	.804
	.2	.410	.655	.920	.815	.885	.850	.801	.767	.714	.613	.787	.833	.836	.731
	.25	.316	.652	.897	.735	.843	.789	.755	.711	.630	.525	.775	.762	.808	.657
	.3	.240	.648	.864	.648	.792	.720	.702	.654	.546	.444	.756	.684	.774	.582
.15	0	.999	.522	.890	.988	.923	.956	.847	.942	.991	.994	.706	.972	.815	.975
	.05	.814	.522	.890	.974	.918	.946	.840	.892	.934	.894	.706	.960	.813	.920
	.1	.656	.522	.887	.936	.903	.919	.820	.841	.866	.796	.704	.928	.805	.858
	.15	.521	.521	.879	.879	.879	.879	.789	.789	.789	.700	.700	.879	.790	.789
	.2	.409	.520	.863	.807	.845	.826	.750	.736	.708	.608	.692	.817	.769	.717
	.25	.316	.518	.840	.726	.802	.764	.703	.680	.624	.521	.679	.745	.741	.643
	.3	.240	.514	.807	.640	.751	.695	.650	.623	.540	.440	.660	.668	.707	.568
.2	0	.998	.410	.819	.973	.870	.922	.794	.902	.979	.986	.614	.947	.742	.954
	.05	.813	.410	.819	.959	.865	.912	.786	.852	.922	.886	.614	.935	.740	.899
	.1	.655	.410	.815	.920	.850	.885	.766	.802	.854	.788	.613	.903	.731	.836
	.15	.520	.409	.807	.863	.826	.845	.736	.750	.778	.692	.608	.854	.717	.768
	.2	.408	.408	.792	.792	.792	.792	.696	.696	.696	.600	.600	.792	.696	.696
	.25	.315	.406	.768	.711	.749	.730	.649	.641	.612	.513	.587	.721	.668	.622
	.3	.238	.402	.735	.625	.699	.662	.597	.584	.528	.431	.569	.643	.633	.547
.25	0	.996	.316	.738	.949	.809	.879	.738	.855	.961	.973	.527	.914	.668	.926
	.05	.811	.316	.738	.935	.804	.869	.731	.805	.904	.873	.527	.902	.666	.871
	.1	.652	.316	.735	.897	.789	.843	.711	.755	.836	.775	.525	.870	.657	.809
	.15	.518	.316	.726	.840	.764	.802	.680	.703	.759	.679	.521	.821	.643	.740
	.2	.406	.315	.711	.768	.730	.749	.641	.649	.678	.587	.513	.759	.622	.668
	.25	.312	.312	.688	.688	.688	.688	.594	.594	.594	.500	.500	.688	.594	.594
	.3	.236	.308	.655	.601	.637	.619	.541	.537	.510	.419	.481	.610	.559	.519
.3	0	.992	.240	.652	.916	.740	.828	.681	.803	.935	.954	.446	.872	.593	.891
	.05	.806	.240	.651	.902	.735	.819	.674	.753	.878	.854	.446	.860	.590	.836
	.1	.648	.240	.648	.864	.720	.792	.654	.702	.810	.756	.444	.828	.582	.774
	.15	.514	.240	.640	.807	.695	.751	.623	.650	.734	.660	.440	.779	.567	.706
	.2	.402	.238	.625	.736	.662	.699	.583	.597	.652	.569	.431	.717	.546	.634
	.25	.308	.236	.601	.655	.619	.637	.537	.541	.568	.481	.419	.646	.518	.559
	.3	.232	.232	.568	.568	.568	.568	.484	.484	.484	.400	.400	.568	.484	.484

Table 13. Probability of Early Function

n = 4

e	d	lslsl	lplplpl-1	lplplpl-2	lplplpl-3	(lpl)s(lpl)	(lsl)p(lsl)	(lslsl)pl	(lplpl)sl-1	(lplpl)sl-2	(lpl)slsl	(lsl)plpl-1	(lsl)plpl-2	lp[ls(lpl)]	ls[lp(lsl)]
0		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.05		.000	.185	.014	.000	.010	.005	.050	.007	.000	.000	.100	.003	.055	.003
.1		.000	.344	.052	.004	.036	.020	.101	.027	.003	.002	.198	.012	.117	.011
.15		.001	.478	.110	.012	.077	.044	.153	.058	.009	.006	.294	.028	.185	.025
.2		.002	.590	.181	.027	.130	.078	.206	.098	.021	.014	.386	.053	.258	.046
.25		.004	.684	.262	.051	.191	.121	.262	.145	.039	.027	.473	.086	.332	.074
.3		.008	.760	.348	.084	.260	.172	.319	.197	.065	.046	.554	.128	.407	.109

Table 14. Probability of Dudding

n = 4

e	d	lslsl	lplplpl-1	lplplpl-2	lplplpl-3	(lpl)s(lpl)	(lsl)p(lsl)	(lslsl)pl	(lplpl)sl-1	(lplpl)sl-2	(lpl)slsl	(lsl)plpl-1	(lsl)plpl-2	lp[ls(lpl)]	ls[lp(lsl)]
	0	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	.05	.185	.000	.000	.014	.005	.010	.007	.050	.057	.100	.000	.012	.003	.055
	.1	.344	.000	.004	.052	.020	.036	.027	.101	.125	.198	.002	.044	.011	.117
	.15	.478	.001	.012	.110	.044	.077	.058	.153	.202	.294	.006	.093	.025	.185
	.2	.590	.002	.027	.181	.078	.130	.098	.206	.283	.385	.014	.155	.046	.258
	.25	.684	.004	.051	.262	.121	.191	.145	.262	.367	.473	.027	.227	.074	.332
	.3	.760	.008	.084	.348	.172	.260	.197	.319	.451	.554	.046	.304	.109	.407

Table 15. Probability of Proper Function

n = 5

e	d	1s1s1s1s1	1p1p1p1p1-1	1p1p1p1p1-2	1p1p1p1p1-3	1p1p1p1p1-4	1s(1p1p1p1)	1p(1s1s1s1)	(1p1p1)s(1p1)	(1s1s1)p(1s1)	[(1s1)p1]s(1p1)	[(1p1)s1]p(1s1)	1s(1p1)s(1p1)	1p(1s1)p(1s1)	(1s1)p1p1p1	(1p1)s1s1s1	1s(1p1p1)s1	1p(1s1s1)p1
0	0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	.05	.774	1.000	1.000	.999	.977	.950	.991	.997	.986	.993	.995	.945	1.000	1.000	.855	.902	1.000
	.1	.590	1.000	1.000	.991	.919	.900	.966	.989	.949	.971	.979	.882	.996	1.000	.722	.809	.997
	.15	.444	1.000	.998	.973	.835	.850	.928	.974	.893	.937	.953	.812	.988	.999	.600	.720	.991
	.2	.328	1.000	.993	.942	.737	.799	.882	.952	.824	.891	.916	.737	.974	.997	.492	.635	.980
	.25	.237	.999	.984	.896	.633	.747	.829	.923	.747	.835	.870	.659	.952	.993	.396	.554	.964
	.3	.168	.998	.969	.837	.528	.694	.772	.885	.665	.771	.815	.580	.922	.986	.312	.477	.941
.05	0	1.000	.774	.977	.999	1.000	.991	.950	.986	.997	.995	.993	1.000	.945	.855	1.000	1.000	.902
	.05	.774	.774	.977	.998	.977	.941	.941	.983	.983	.988	.988	.945	.945	.855	.855	.902	.902
	.1	.590	.774	.977	.990	.919	.891	.916	.975	.946	.966	.972	.882	.942	.855	.722	.809	.900
	.15	.444	.774	.975	.972	.835	.840	.878	.960	.890	.932	.946	.812	.934	.854	.600	.720	.894
	.2	.328	.773	.971	.941	.737	.789	.832	.938	.822	.886	.909	.737	.919	.852	.492	.635	.883
	.25	.237	.773	.962	.895	.633	.738	.779	.909	.744	.830	.863	.659	.897	.848	.395	.553	.866
	.3	.168	.771	.947	.836	.528	.685	.722	.872	.662	.766	.808	.579	.867	.841	.312	.476	.843
.1	0	1.000	.590	.919	.991	1.000	.966	.900	.949	.989	.979	.971	.996	.882	.722	1.000	.997	.809
	.05	.774	.590	.919	.990	.977	.916	.891	.946	.975	.972	.966	.942	.882	.722	.855	.900	.809
	.1	.590	.590	.918	.983	.918	.866	.866	.938	.938	.950	.950	.878	.878	.722	.722	.806	.806
	.15	.444	.590	.916	.965	.835	.815	.828	.923	.882	.916	.924	.809	.871	.721	.600	.717	.801
	.2	.328	.590	.912	.934	.737	.764	.782	.901	.813	.870	.888	.734	.856	.719	.491	.632	.790
	.25	.237	.590	.903	.888	.632	.713	.729	.871	.736	.814	.841	.656	.834	.715	.395	.551	.773
	.3	.168	.588	.888	.828	.528	.660	.672	.834	.654	.750	.786	.576	.804	.708	.312	.474	.750
.15	0	1.000	.444	.835	.973	.998	.928	.850	.893	.974	.953	.937	.988	.812	.600	.999	.991	.720
	.05	.774	.444	.835	.972	.975	.878	.840	.890	.960	.946	.932	.934	.812	.600	.854	.894	.720
	.1	.590	.444	.835	.965	.916	.828	.815	.882	.923	.924	.916	.871	.809	.600	.721	.801	.717
	.15	.444	.444	.833	.947	.833	.778	.778	.867	.867	.890	.890	.801	.801	.599	.599	.711	.711
	.2	.328	.443	.828	.915	.735	.727	.731	.845	.799	.844	.853	.726	.786	.597	.491	.626	.701
	.25	.237	.443	.820	.870	.631	.675	.679	.816	.721	.788	.807	.648	.764	.593	.395	.545	.684
	.3	.168	.441	.804	.810	.526	.623	.622	.778	.639	.724	.752	.568	.734	.587	.311	.468	.661
.2	0	1.000	.328	.737	.942	.993	.882	.799	.824	.952	.916	.891	.974	.737	.492	.997	.980	.635
	.05	.773	.328	.737	.941	.971	.832	.789	.822	.938	.909	.886	.919	.737	.492	.852	.883	.635
	.1	.590	.328	.737	.934	.912	.782	.764	.813	.901	.888	.870	.856	.734	.491	.719	.790	.632
	.15	.443	.328	.735	.915	.828	.731	.727	.799	.845	.853	.844	.786	.726	.491	.597	.701	.626
	.2	.327	.327	.731	.884	.731	.681	.681	.777	.777	.807	.807	.711	.711	.489	.489	.615	.615
	.25	.237	.327	.722	.839	.626	.629	.628	.747	.699	.751	.761	.633	.689	.485	.393	.534	.599
	.3	.168	.325	.706	.779	.522	.576	.571	.710	.617	.687	.706	.554	.659	.478	.309	.457	.576
.25	0	.999	.237	.633	.896	.984	.829	.747	.747	.923	.870	.835	.952	.659	.396	.993	.964	.554
	.05	.773	.237	.633	.895	.962	.779	.738	.744	.909	.863	.830	.897	.659	.395	.848	.866	.553
	.1	.590	.237	.632	.888	.903	.729	.713	.736	.871	.841	.814	.834	.656	.395	.715	.773	.551
	.15	.443	.237	.631	.870	.820	.679	.675	.721	.816	.807	.788	.764	.648	.395	.593	.684	.545
	.2	.327	.237	.626	.839	.722	.628	.629	.699	.747	.761	.751	.689	.633	.393	.485	.599	.534
	.25	.236	.236	.617	.793	.617	.576	.576	.670	.670	.705	.705	.611	.611	.389	.389	.518	.518
	.3	.167	.235	.602	.733	.513	.523	.519	.633	.588	.641	.650	.532	.581	.382	.305	.441	.495
.3	0	.998	.168	.528	.837	.969	.772	.694	.665	.885	.815	.771	.922	.580	.312	.986	.941	.477
	.05	.771	.168	.528	.836	.947	.722	.685	.662	.872	.808	.766	.867	.579	.312	.841	.843	.476
	.1	.588	.168	.528	.828	.888	.672	.660	.654	.834	.786	.750	.804	.576	.312	.708	.750	.474
	.15	.441	.168	.526	.810	.804	.622	.623	.639	.778	.752	.724	.734	.568	.311	.587	.661	.468
	.2	.325	.168	.522	.779	.706	.571	.576	.617	.710	.706	.687	.659	.554	.309	.478	.576	.457
	.25	.235	.167	.513	.733	.602	.519	.523	.588	.633	.650	.641	.581	.532	.305	.382	.495	.441
	.3	.166	.166	.497	.674	.497	.466	.466	.550	.550	.586	.586	.502	.502	.298	.298	.418	.418

2

ınction

⟨1p1⟩s⟨1p1⟩	(1s1s1)p(1s1)	[(1s1)p1]s(1p1)	[(1p1)s1]p(1s1)	1s(1p1)s(1p1)	1p(1s1)p(1s1)	(1s1)p1p1p1	(1p1)s1s1s1	1s(1p1p1)s1	1p(1s1s1)p1	[(1s1)p1]s1s1	[(1p1)s1]p1p1	[(1s1)p1p1]s1	[(1p1)s1s1]p1	[(1p1p1)s1]p1	[(1s1s1)p1]s1	[(1p1)s(1p1)]p1	[(1s1)p(1s1)]s1	{1s[(1s1)p1]}p1	{1p[(1p1)s1]}s1
00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
97	.986	.993	.995	.945	1.000	1.000	.855	.902	1.000	.898	1.000	.950	.995	.997	.943	1.000	.941	.997	.948
89	.949	.971	.979	.882	.996	1.000	.722	.809	.997	.795	.999	.898	.980	.990	.876	.998	.868	.988	.890
74	.893	.937	.953	.812	.988	.999	.600	.720	.991	.692	.996	.845	.956	.977	.801	.993	.785	.972	.828
52	.824	.891	.916	.737	.974	.997	.492	.635	.980	.594	.991	.788	.923	.959	.722	.984	.696	.948	.763
23	.747	.835	.870	.659	.952	.993	.396	.554	.964	.501	.981	.729	.882	.935	.642	.970	.606	.917	.694
85	.665	.771	.815	.580	.922	.986	.312	.477	.941	.415	.967	.668	.834	.904	.562	.948	.518	.878	.624
86	.997	.995	.993	1.000	.945	.855	1.000	1.000	.902	1.000	.898	.995	.950	.943	.997	.941	1.000	.948	.997
83	.983	.988	.988	.945	.945	.855	.855	.902	.902	.898	.898	.945	.945	.941	.941	.941	.941	.945	.945
75	.946	.966	.972	.882	.942	.855	.722	.809	.900	.794	.897	.893	.930	.933	.873	.939	.867	.936	.887
60	.890	.932	.946	.812	.934	.854	.600	.720	.894	.692	.894	.840	.906	.920	.798	,934	.784	.920	.826
38	.822	.886	.909	.737	.919	.852	.492	.635	.883	.594	.889	.783	.873	.902	.719	.925	.696	.896	.760
09	.744	.830	.863	.659	.897	.848	.395	.553	.866	.501	.880	.725	.832	.878	.639	.911	.606	.865	.692
72	.662	.766	.808	.579	.867	.841	.312	.476	.843	.415	.865	.663	.784	.848	.560	.889	.518	.825	.621
49	.989	.979	.971	.996	.882	.722	1.000	.997	.809	.999	.795	.980	.898	.876	.990	.868	.998	.890	.988
46	.975	.972	[illegible]	.942	.882	.722	.855	.900	.809	.897	.794	.930	.893	.873	.933	.867	.939	.887	.936
38	.938	.950	[illegible]	.878	.878	.722	.722	.806	.806	.794	.794	.878	.878	.866	.866	.866	.866	.878	.878
23	.882	.916	[illegible]	.809	.871	.721	.600	.717	.801	.691	.791	.825	.854	.853	.791	.861	.783	.862	.817
01	.813	.870	[illegible]	.734	.856	.719	.491	.632	.790	.593	.785	.769	.821	.834	.712	.852	.694	.839	.751
71	.736	.814	.841	.656	.834	.715	.395	.551	.773	.500	.776	.710	.780	.810	.632	.837	.604	.807	.683
34	.654	.750	.786	.576	.804	.708	.312	.474	.750	.414	.762	.648	.732	.780	.552	.816	.516	.768	.612
93	.974	.953	.937	.988	.812	.600	.999	.991	.720	.996	.692	.956	.845	.801	.977	.785	.993	.828	.972
90	.960	.946	.932	.934	.812	.600	.854	.894	.720	.894	.692	.906	.840	.798	.920	.784	.934	.826	.920
82	.923	.924	.916	.871	.809	.600	.721	.801	.717	.791	.691	.854	.825	.791	.853	.783	.861	.817	.862
67	.867	.890	.890	.801	.801	.599	.599	.711	.711	.689	.689	.801	.801	.778	.778	.778	.778	.801	.801
45	.799	.844	.853	.726	.786	.597	.491	.626	.701	.590	.683	.744	.768	.760	.699	.769	.690	.777	.735
16	.721	.788	.807	.648	.764	.593	.395	.545	.684	.497	.674	.685	.727	.735	.619	.754	.600	.745	.667
78	.639	.724	.752	.568	.734	.587	.311	.468	.661	.411	.660	.624	.678	.705	.539	.733	.511	.706	.596
24	.952	.916	.891	.974	.737	.492	.997	.980	.635	.991	.594	.923	.788	.722	.959	.696	.984	.763	.948
22	.938	.909	.886	.919	.737	.492	.852	.883	.635	.889	.594	.873	.783	.719	.902	.696	.925	.760	.896
13	.901	.888	.870	.856	.734	.491	.719	.790	.632	.785	.593	.821	.769	.712	.834	.694	.852	.751	.839
799	.845	.853	.844	.786	.726	.491	.597	.701	.626	.683	.590	.768	.744	.699	.760	.690	.769	.735	.777
777	.777	.807	.807	.711	.711	.489	.489	.615	.615	.585	.585	.711	.711	.681	.681	.681	.681	.711	.711
747	.699	.751	.761	.633	.689	.485	.393	.534	.599	.492	.575	.652	.670	.656	.600	.666	.591	.680	.643
710	.617	.687	.706	.554	.659	.478	.309	.457	.576	.406	.561	.591	.622	.626	.521	.645	.502	.641	.572
747	.923	.870	.835	.952	.659	.396	.993	.964	.554	.981	.501	.882	.729	.642	.935	.606	.970	.694	.917
744	.909	.863	.830	.897	.659	.395	.848	.866	.553	.880	.501	.832	.725	.639	.878	.606	.911	.692	.865
736	.871	.841	.814	.834	.656	.395	.715	.773	.551	.776	.500	.780	.710	.632	.810	.604	.837	.683	.807
721	.816	.807	.788	.764	.648	.395	.593	.684	.545	.674	.497	.727	.685	.619	.735	.600	.754	.667	.745
699	.747	.761	.751	.689	.633	.393	.485	.599	.534	.575	.492	.670	.652	.600	.656	.591	.666	.643	.680
670	.670	.705	.705	.611	.611	.389	.389	.518	.518	.482	.482	.611	.611	.576	.576	.576	.576	.611	.611
633	.588	.641	.650	.532	.581	.382	.305	.441	.495	.396	.468	.550	.563	.546	.497	.555	.488	.572	.541
665	.885	.815	.771	.922	.580	.312	.986	.941	.477	.967	.415	.834	.668	.562	.904	.518	.948	.624	.878
662	.872	.808	.766	.867	.579	.312	.841	.843	.476	.865	.415	.784	.663	.560	.848	.518	.889	.621	.825
654	.834	.786	.750	.804	.576	.312	.708	.750	.474	.762	.414	.732	.648	.552	.780	.516	.816	.612	.768
639	.778	.752	.724	.734	.568	.311	.587	.661	.468	.660	.411	.678	.624	.539	.705	.511	.733	.596	.706
617	.710	.706	.687	.659	.554	.309	.478	.576	.457	.561	.406	.622	.591	.521	.626	.502	.645	.572	.641
588	.633	.650	.641	.581	.532	.305	.382	.495	.441	.468	.396	.563	.550	.497	.546	.488	.555	.541	.572
550	.550	.586	.586	.502	.502	.298	.298	.418	.418	.382	.382	.502	.502	.466	.466	.466	.466	.502	.502

Table 16. Probability of Early Function

n = 5

e d	1s1s1s1	1p1p1p1-1	1p1p1p1-2	1p1p1p1-3	1p1p1p1-4	1s(1p1p1)	1p(1s1s1)	(1p1p1)s(1p1)	(1s1s1)p(1s1)	[(1s1)p1]s(1p1)	[(1p1)s1]p(1s1)	1s(1p1)s(1p1)	1p(1s1)p(1s1)	(1s1)p1p1p1	(1p1)s1s1s1	(1p1p1)s1s1	(1s1s1)p1p1	[(1s1)p1]s1s1	[(1p1)s1]p1p1	[(1s1)p1p1]s1	[(1p1)s1s1]p1	[(1p1p1)s1]p1	[(1s1s1)p1]s1	[(1p1)s(1p1)]p1	[(1s1)p(1s1)]s1	{1s[(1s1)p1]}p1	{1p[(1p1)s1]}s1
0	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.05	.000	.226	.023	.001	.000	.009	.050	.014	.003	.005	.007	.000	.055	.145	.000	.000	.098	.000	.102	.005	.050	.057	.003	.059	.000	.052	.003
.1	.000	.410	.081	.009	.000	.034	.100	.051	.011	.021	.029	.004	.118	.278	.000	.003	.191	.001	.205	.020	.102	.124	.010	.132	.002	.110	.012
.15	.000	.556	.165	.027	.002	.072	.150	.107	.026	.047	.063	.012	.188	.400	.001	.009	.280	.004	.308	.044	.155	.199	.023	.215	.007	.172	.028
.2	.000	.672	.263	.058	.007	.118	.201	.176	.048	.084	.109	.026	.263	.508	.003	.020	.365	.009	.406	.077	.212	.278	.041	.304	.016	.237	.052
.25	.001	.763	.367	.104	.016	.171	.253	.253	.077	.130	.165	.048	.341	.604	.007	.036	.446	.019	.499	.118	.271	.358	.065	.394	.030	.306	.083
.3	.002	.832	.472	.163	.031	.228	.306	.335	.115	.185	.229	.078	.420	.688	.014	.059	.523	.033	.585	.166	.332	.438	.096	.482	.052	.376	.122

Table 17. Probability of Dudding

n = 5

0	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.05	.226	.000	.000	.001	.023	.050	.009	.003	.014	.007	.005	.055	.000	.000	.145	.098	.000	.102	.000	.050	.005	.003	.057	.000	.059	.003	.052
.1	.410	.000	.000	.009	.081	.100	.034	.011	.051	.029	.021	.118	.004	.000	.278	.191	.003	.205	.001	.102	.020	.010	.124	.002	.132	.012	.110
.15	.556	.000	.002	.027	.165	.150	.072	.026	.107	.063	.047	.188	.012	.001	.400	.280	.009	.308	.004	.155	.044	.023	.199	.007	.215	.028	.172
.2	.672	.000	.007	.058	.263	.201	.118	.048	.176	.109	.084	.263	.026	.003	.508	.365	.020	.406	.009	.212	.077	.041	.278	.016	.304	.052	.237
.25	.763	.001	.016	.104	.367	.253	.171	.077	.253	.165	.130	.341	.048	.007	.604	.446	.036	.499	.019	.271	.118	.065	.358	.030	.394	.083	.306
.3	.832	.002	.031	.163	.472	.306	.228	.115	.335	.229	.185	.420	.078	.014	.688	.523	.059	.585	.033	.332	.166	.096	.438	.052	.482	.122	.376

Table 18. Average System Proper Function Probabilities

W_E	W_D	$0\leq e\leq .1$ $0\leq d\leq .1$ $\lvert p\rvert-1$	 $\lvert s\rvert$	$0\leq e\leq .15$ $0\leq d\leq .15$ $\lvert p\rvert-1$	 $\lvert s\rvert$	$0\leq e\leq .2$ $0\leq d\leq .2$ $\lvert p\rvert-1$	 $\lvert s\rvert$	$0\leq e\leq .1$ $0\leq d\leq .2$ $\lvert p\rvert-1$	 $\lvert s\rvert$	$0\leq e\leq .2$ $0\leq d\leq .1$ $\lvert p\rvert-1$	 $\lvert s\rvert$
1	1	.899	.899	.850	.850	.800	.800	.889	.810	.810	.889
1.5	.5	.854	.946	.783	.916	.642	.745	.848	.901	.901	.848
1.33	.67	.869	.930	.806	.893	.661	.719	.862	.870	.870	.862
.67	1.33	.930	.869	.893	.806	.719	.661	.870	.862	.862	.870
.5	1.5	.946	.854	.916	.783	.745	.642	.901	.848	.848	.901

Table 19. Average System Proper Function Probabilities

n = 3

	$0 \leq e \leq .1$, $0 \leq d \leq .1$	$0 \leq e \leq .15$, $0 \leq d \leq .15$	$0 \leq e \leq .2$, $0 \leq d \leq .2$	$0 \leq e \leq .1$, $0 \leq d \leq .2$	$0 \leq e \leq .2$, $0 \leq d \leq .1$
$W_E = W_D = 1$					
1s1s1	.862	.799	.740	.742	.860
1p1p1-1	.862	.799	.740	.860	.742
1p1p1-2	.977	.952	.920	.948	.948
(1p1)s1	.938	.901	.860	.880	.919
(1s1)p1	.938	.901	.860	.919	.880
$W_E=1.5$ $W_D=0.5$					
1s1s1	.931	.898	.867	.871	.927
1p1p1-1	.793	.700	.612	.792	.614
1p1p1-2	.977	.952	.920	.962	.934
(1p1)s1	.961	.934	.902	.932	.932
(1s1)p1	.915	.868	.817	.906	.827
$W_E=1.33$ $W_D=0.67$					
1s1s1	.907	.865	.824	.827	.904
1p1p1-1	.816	.733	.656	.815	.657
1p1p1-2	.977	.952	.920	.958	.939
(1p1)s1	.953	.923	.888	.914	.927
(1s1)p1	.923	.879	.832	.910	.845

	$0 \leq e \leq .1$, $0 \leq d \leq .1$	$0 \leq e \leq .15$, $0 \leq d \leq .15$	$0 \leq e \leq .2$, $0 \leq d \leq .2$	$0 \leq e \leq .1$, $0 \leq d \leq .2$	$0 \leq e \leq .2$, $0 \leq d \leq .1$
$W_E=0.67$ $W_D=1.33$					
1s1s1	.816	.733	.656	.657	.815
1p1p1-1	.907	.865	.824	.904	.827
1p1p1-2	.977	.952	.920	.939	.958
(1p1)s1	.923	.879	.832	.845	.910
(1s1)p1	.953	.923	.888	.927	.914
$W_E=0.5$ $W_D=1.5$					
1s1s1	.793	.700	.612	.614	.792
1p1p1-1	.931	.898	.867	.927	.871
1p1p1-2	.977	.952	.920	.934	.962
(1p1)s1	.915	.868	.817	.827	.906
(1s1)p1	.961	.934	.902	.932	.932

Table 20. Average System Proper Function Probabilities

n = 4

$W_E=W_D=1$	$0\le e\le .1$ $0\le d\le .1$	$0\le e\le .15$ $0\le d\le .15$	$0\le e\le .2$ $0\le d\le .2$	$0\le e\le .1$ $0\le d\le .2$	$0\le e\le .2$ $0\le d\le .1$
1s1s1s1	.824	.748	.680	.680	.823
1p1p1p1-1	.824	.748	.680	.823	.680
1p1p1p1-2	.977	.952	.920	.969	.927
1p1p1p1-3	.976	.952	.920	.927	.969
(1p1)s(1p1)	.976	.952	.920	.955	.941
(1s1)p(1s1)	.976	.952	.920	.941	.955
(1s1s1)p1	.938	.901	.860	.912	.887
(1p1p1)s1-1	.938	.901	.860	.887	.912
(1p1p1)s1-2	.938	.901	.860	.866	.933
(1p1)s1s1	.900	.850	.800	.804	.896
(1s1)p1p1-1	.900	.850	.800	.896	.804
(1s1)p1p1-2	.977	.952	.920	.934	.962
1p [1s(1p1)]	.938	.901	.860	.926	.873
1s [1p(1s1)]	.938	.901	.860	.873	.926

$W_E=1.5$ $W_D=0.5$	$0\le e\le .1$ $0\le d\le .1$	$0\le e\le .15$ $0\le d\le .15$	$0\le e\le .2$ $0\le d\le .2$	$0\le e\le .1$ $0\le d\le .2$	$0\le e\le .2$ $0\le d\le .1$
1s1s1s1	.912	.874	.840	.840	.911
1p1p1p1-1	.735	.622	.520	.735	.521
1p1p1p1-2	.966	.932	.889	.963	.892
1p1p1p1-3	.987	.972	.951	.962	.976
(1p1)s(1p1)	.973	.945	.910	.962	.920
(1s1)p(1s1)	.980	.959	.930	.962	.948
(1s1s1)p1	.919	.875	.828	.906	.841
(1p1p1)s1-1	.958	.927	.892	.932	.918
(1p1p1)s1-2	.968	.947	.923	.932	.960
(1p1)s1s1	.949	.923	.895	.901	.944
(1s1)p1p1-1	.851	.777	.705	.849	.706
(1s1)p1p1-2	.983	.965	.941	.962	.962
1p [1s(1p1)]	.912	.861	.807	.906	.813
1s [1p(1s1)]	.965	.941	.913	.932	.946

$W_E=1.33$ $W_D=0.67$	$0\le e\le .1$ $0\le d\le .1$	$0\le e\le .15$ $0\le d\le .15$	$0\le e\le .2$ $0\le d\le .2$	$0\le e\le .1$ $0\le d\le .2$	$0\le e\le .2$ $0\le d\le .1$
1s1s1s1	.882	.831	.785	.786	.881
1p1p1p1-1	.765	.665	.575	.765	.575
1p1p1p1-2	.970	.939	.899	.965	.904
1p1p1p1-3	.983	.965	.941	.950	.974
(1p1)s(1p1)	.974	.948	.913	.960	.927
(1s1)p(1s1)	.979	.956	.927	.955	.951
(1s1s1)p1	.925	.884	.839	.908	.857
(1p1p1)s1-1	.951	.918	.881	.916	.916
(1p1p1)s1-2	.958	.932	.902	.909	.951
(1p1)s1s1	.933	.898	.863	.868	.527
(1s1)p1p1-1	.867	.802	.737	.865	.740
(1s1)p1p1-2	.981	.961	.934	.953	.962
1p [1s(1p1)]	.921	.875	.825	.912	.833
1s [1p(1s1)]	.956	.927	.895	.912	.939

$W_E=0.67$ $W_D=1.33$	$0\le e\le .1$ $0\le d\le .1$	$0\le e\le .15$ $0\le d\le .15$	$0\le e\le .2$ $0\le d\le .2$	$0\le e\le .1$ $0\le d\le .2$	$0\le e\le .2$ $0\le d\le .1$
1s1s1s1	.765	.665	.575	.575	.765
1p1p1p1-1	.882	.831	.785	.881	.786
1p1p1p1-2	.983	.965	.941	.974	.950
1p1p1p1-3	.970	.939	.899	.904	.965
(1p1)s(1p1)	.979	.956	.927	.951	.955
(1s1)p(1s1)	.974	.948	.913	.927	.960
(1s1s1)p1	.951	.918	.881	.916	.916
(1p1p1)s1-1	.925	.884	.839	.857	.908
(1p1p1)s1-2	.919	.870	.818	.822	.915
(1p1)s1s1	.867	.802	.737	.740	.865
(1s1)p1p1-1	.933	.898	.863	.927	.868
(1s1)p1p1-2	.972	.943	.906	.916	.962
1p [1s(1p1)]	.956	.927	.895	.939	.912
1s [1p(1s1)]	.921	.875	.825	.833	.912

$W_E=0.5$ $W_D=1.5$	$0\le e\le .1$ $0\le d\le .1$	$0\le e\le .15$ $0\le d\le .15$	$0\le e\le .2$ $0\le d\le .2$	$0\le e\le .1$ $0\le d\le .2$	$0\le e\le .2$ $0\le d\le .1$
1s1s1s1	.735	.622	.520	.521	.735
1p1p1p1-1	.912	.874	.840	.911	.840
1p1p1p1-2	.987	.972	.951	.976	.962
1p1p1p1-3	.966	.932	.889	.892	.963
(1p1)s(1p1)	.980	.959	.930	.948	.962
(1s1)p(1s1)	.973	.945	.910	.920	.962
(1s1s1)p1	.958	.927	.892	.918	.932
(1p1p1)s1-1	.919	.875	.828	.841	.906
(1p1p1)s1-2	.908	.855	.797	.799	.906
(1p1)s1s1	.851	.777	.705	.706	.849
(1s1)p1p1-1	.949	.923	.895	.944	.901
(1s1)p1p1-2	.970	.939	.899	.906	.962
1p [1s(1p1)]	.965	.941	.913	.946	.932
1s [1p(1s1)]	.912	.861	.807	.813	.906

Table 21. Average System Proper Function Probabilities

$n = 5$

	$0 \leq e \leq .1$, $0 \leq d \leq .1$	$0 \leq e \leq .15$, $0 \leq d \leq .15$	$0 \leq e \leq .2$, $0 \leq d \leq .2$	$0 \leq e \leq .1$, $0 \leq d \leq .2$	$0 \leq e \leq .2$, $0 \leq d \leq .1$	$0 \leq e \leq .1$, $0 \leq d \leq .1$	$0 \leq e \leq .15$, $0 \leq d \leq .15$	$0 \leq e \leq .2$, $0 \leq d \leq .2$
	$W_E = W_D = 1$					$W_E = 1.5$ $W_D = 0.5$		
1s1s1s1s1	.788	.702	.627	.627	.788	.894	.851	.813
1p1p1p1p1-1	.788	.702	.627	.788	.627	.682	.553	.441
1p1p1p1p1-2	.965	.932	.892	.963	.894	.948	.899	.840
1p1p1p1p1-3	.994	.982	.962	.978	.978	.994	.982	.962
1p1p1p1p1-4	.965	.932	.892	.894	.963	.982	.965	.944
1s(1p1p1p1)	.935	.896	.853	.885	.903	.953	.919	.880
1p(1s1s1s1)	.935	.896	.853	.903	.885	.918	.873	.826
(1p1p1)s(1p1)	.974	.947	.913	.961	.926	.965	.930	.887
(1s1s1)p(1s1)	.974	.947	.913	.926	.961	.982	.964	.939
[(1s1)p1]s(1p1)	.979	.957	.927	.950	.957	.981	.960	.932
[(1p1)s1]p(1s1)	.979	.957	.927	.957	.950	.978	.954	.922
1s(1p1)s(1p1)	.941	.906	.867	.874	.934	.969	.949	.925
1p(1s1)p(1s1)	.941	.906	.867	.934	.874	.913	.863	.809
(1s1)p1p1p1	.859	.794	.733	.858	.734	.788	.691	.600
(1p1)s1s1s1	.859	.794	.733	.734	.858	.929	.897	.866
(1p1p1)s1s1	.903	.855	.807	.812	.898	.950	.925	.897
(1s1s1)p1p1	.903	.855	.807	.898	.812	.855	.785	.717
[(1s1)p1]s1s1	.897	.845	.793	.795	.895	.948	.921	.894
[(1p1)s1]p1p1	.897	.845	.793	.895	.795	.846	.769	.692
[(1s1)p1p1]s1	.941	.906	.867	.888	.920	.962	.936	.904
[(1p1)s1s1]p1	.941	.906	.867	.920	.888	.920	.876	.830
[(1p1p1)s1]p1	.935	.896	.853	.924	.864	.907	.853	.795
[(1s1s1)p1]s1	.935	.896	.853	.864	.924	.964	.939	.911
[(1p1)s(1p1)]p1	.935	.896	.853	.931	.857	.904	.846	.784
[(1s1)p(1s1)]s1	.935	.896	.853	.857	.931	.967	.946	.922
{1s[(1s1)p1]} p1	.941	.906	.867	.927	.881	.916	.870	.819
{1p[(1p1)s1]} s1	.941	.906	.867	.881	.927	.966	.942	.915

	$W_E = 0.67$ $W_D = 1.33$					$W_E = 0.5$ $W_D = 1.5$		
1s1s1s1s1	.718	.604	.504	.504	.718	.682	.553	.441
1p1p1p1p1-1	.858	.800	.750	.858	.750	.894	.851	.813
1p1p1p1p1-2	.977	.954	.926	.974	.929	.982	.965	.944
1p1p1p1p1-3	.994	.982	.962	.973	.983	.994	.982	.962
1p1p1p1p1-4	.954	.910	.857	.858	.953	.948	.899	.840
1s(1p1p1p1)	.924	.881	.835	.857	.902	.918	.873	.826
1p(1s1s1s1)	.947	.911	.871	.904	.913	.953	.919	.880
(1p1p1)s(1p1)	.979	.958	.930	.962	.947	.982	.964	.939
(1s1s1)p(1s1)	.968	.936	.896	.904	.959	.965	.930	.887
[(1s1)p1]s(1p1)	.978	.955	.924	.939	.963	.978	.954	.922
[(1p1)s1]p(1s1)	.980	.959	.930	.950	.961	.981	.960	.932
1s(1p1)s(1p1)	.923	.878	.829	.833	.918	.913	.863	.809
1p(1s1)p(1s1)	.960	.934	.905	.950	.915	.969	.949	.925
(1s1)p1p1p1	.905	.862	.821	.904	.822	.929	.897	.866
(1p1)s1s1s1	.812	.726	.645	.646	.812	.788	.691	.600
(1p1p1)s1s1	.871	.809	.748	.751	.868	.855	.785	.717
(1s1s1)p1p1	.934	.901	.867	.927	.874	.950	.925	.897
[(1s1)p1]s1s1	.863	.795	.727	.728	.862	.846	.769	.692
[(1p1)s1]p1p1	.931	.895	.859	.928	.863	.948	.921	.894
[(1s1)p1p1]s1	.927	.886	.842	.856	.913	.920	.876	.830
[(1p1)s1s1]p1	.955	.926	.892	.927	.919	.962	.936	.904
[(1p1p1)s1]p1	.954	.924	.891	.939	.906	.964	.939	.911
[(1s1s1)p1]s1	.917	.868	.815	.822	.909	.907	.853	.795
[(1p1)s(1p1)]p1	.956	.929	.898	.951	.904	.967	.946	.922
[(1s1)p(1s1)]s1	.915	.863	.808	.810	.912	.904	.846	.784
{1s[(1s1)p1]}p1	.957	.930	.899	.939	.917	.966	.942	.915
{1p[(1p1)s1]}s1	.925	.882	.836	.845	.915	.916	.870	.819

tion Probabilities

$\le .1$ / $\le .2$	$0 \le e \le .2$ / $0 \le d \le .1$	$0 \le e \le .1$ / $0 \le d \le .1$	$0 \le e \le .15$ / $0 \le d \le .15$	$0 \le e \le .2$ / $0 \le d \le .2$	$0 \le e \le .1$ / $0 \le d \le .2$	$0 \le e \le .2$ / $0 \le d \le .1$	$0 \le e \le .1$ / $0 \le d \le .1$	$0 \le e \le .15$ / $0 \le d \le .15$	$0 \le e \le .2$ / $0 \le d \le .2$	$0 \le e \le .1$ / $0 \le d \le .2$	$0 \le e \le .2$ / $0 \le d \le .1$
		$W_E=1.5$ $W_D=0.5$					$W_E=1.33$ $W_D=0.67$				
'	.788	.894	.851	.813	.814	.894	.858	.800	.750	.750	.858
3	.627	.682	.553	.441	.682	.441	.718	.604	.504	.718	.504
3	.894	.948	.899	.840	.947	.840	.954	.910	.857	.953	.858
3	.978	.994	.982	.962	.986	.970	.994	.982	.962	.983	.973
4	.963	.982	.965	.944	.947	.980	.977	.954	.926	.929	.974
5	.903	.953	.919	.880	.928	.905	.947	.911	.871	.913	.904
3	.885	.918	.873	.826	.902	.842	.924	.881	.835	.902	.857
1	.926	.965	.930	.887	.959	.893	.968	.936	.896	.959	.904
5	.961	.982	.964	.939	.958	.963	.979	.958	.930	.947	.962
0	.957	.981	.960	.932	.966	.947	.980	.959	.930	.961	.950
7	.950	.978	.954	.922	.966	.933	.978	.955	.924	.963	.939
4	.934	.969	.949	.925	.936	.959	.960	.934	.905	.915	.950
4	.874	.913	.863	.809	.910	.812	.923	.878	.829	.918	.833
3	.734	.788	.691	.600	.788	.601	.812	.726	.645	.812	.646
4	.858	.929	.897	.866	.867	.928	.905	.862	.821	.822	.904
2	.898	.950	.925	.897	.905	.943	.934	.901	.867	.874	.927
3	.812	.855	.785	.717	.853	.719	.871	.809	.748	.868	.751
5	.895	.948	.921	.894	.897	.944	.931	.895	.859	.863	.928
5	.795	.846	.769	.692	.845	.694	.863	.795	.727	.862	.728
8	.920	.962	.936	.904	.936	.931	.955	.926	.892	.919	.927
0	.888	.920	.876	.830	.909	.840	.927	.886	.842	.913	.856
4	.864	.907	.853	.795	.902	.800	.917	.868	.815	.909	.822
4	.924	.964	.939	.911	.928	.947	.954	.924	.891	.906	.939
1	.857	.904	.846	.784	.902	.786	.915	.863	.808	.912	.810
7	.931	.967	.946	.922	.928	.961	.956	.929	.898	.904	.951
7	.881	.916	.870	.819	.909	.826	.925	.882	.836	.915	.845
1	.927	.966	.942	.915	.936	.945	.957	.930	.899	.917	.939
		$W_E=0.5$ $W_D=1.5$									
4	.718	.682	.553	.441	.441	.682					
8	.750	.894	.851	.813	.894	.814					
4	.929	.982	.965	.944	.980	.947					
3	.983	.994	.982	.962	.970	.986					
8	.953	.948	.899	.840	.840	.947					
7	.902	.918	.873	.826	.842	.902					
4	.913	.953	.919	.880	.905	.928					
2	.947	.982	.964	.939	.963	.958					
4	.959	.965	.930	.887	.893	.959					
9	.963	.978	.954	.922	.933	.966					
0	.961	.981	.960	.932	.947	.966					
3	.918	.913	.863	.809	.812	.910					
0	.915	.969	.949	.925	.959	.936					
4	.822	.929	.897	.866	.928	.867					
6	.812	.788	.691	.600	.601	.788					
1	.868	.855	.785	.717	.719	.853					
7	.874	.950	.925	.897	.943	.905					
8	.862	.846	.769	.692	.694	.845					
8	.863	.948	.921	.894	.944	.897					
6	.913	.920	.876	.830	.840	.909					
7	.919	.962	.936	.904	.931	.936					
9	.906	.964	.939	.911	.947	.928					
2	.909	.907	.853	.795	.800	.902					
1	.904	.967	.946	.922	.961	.928					
0	.912	.904	.846	.784	.786	.902					
9	.917	.966	.942	.915	.945	.936					
5	.915	.916	.870	.819	.826	.909					

Table 22. Average System Proper Function Probabilities

	$0 \le e \le .1$, $0 \le d \le .1$	$0 \le e \le .15$, $0 \le d \le .15$	$0 \le e \le .2$, $0 \le d \le .2$	$0 \le e \le .1$, $0 \le d \le .2$	$0 \le e \le .2$, $0 \le d \le .1$
$W_E=W_D=1$					
1s1	.899	.850	.800	.810	.889
1p1-1	.899	.850	.800	.889	.810
1p1p1-2	.977	.952	.920	.948	.948
1p1p1p1-2	.977	.952	.920	.969	.927
1p1p1p1-3	.976	.952	.920	.927	.969
1p1p1p1p1-3	.994	.982	.962	.978	.978
$W_E=1.5$ $W_D=0.5$					
1s1	.946	.916	.745	.901	.848
1p1-1	.854	.783	.642	.848	.901
1p1p1-2	.977	.952	.920	.962	.934
1p1p1p1-2	.966	.932	.889	.963	.892
1p1p1p1-3	.987	.972	.951	.962	.976
1p1p1p1p1-3	.994	.982	.962	.986	.970
$W_E=1.33$ $W_D=0.67$					
1s1	.930	.893	.719	.870	.862
1p1-1	.869	.806	.661	.862	.870
1p1p1-2	.977	.952	.920	.958	.939
1p1p1p1-2	.970	.939	.899	.965	.904
1p1p1p1-3	.983	.965	.941	.950	.974
1p1p1p1p1-3	.994	.982	.962	.983	.973

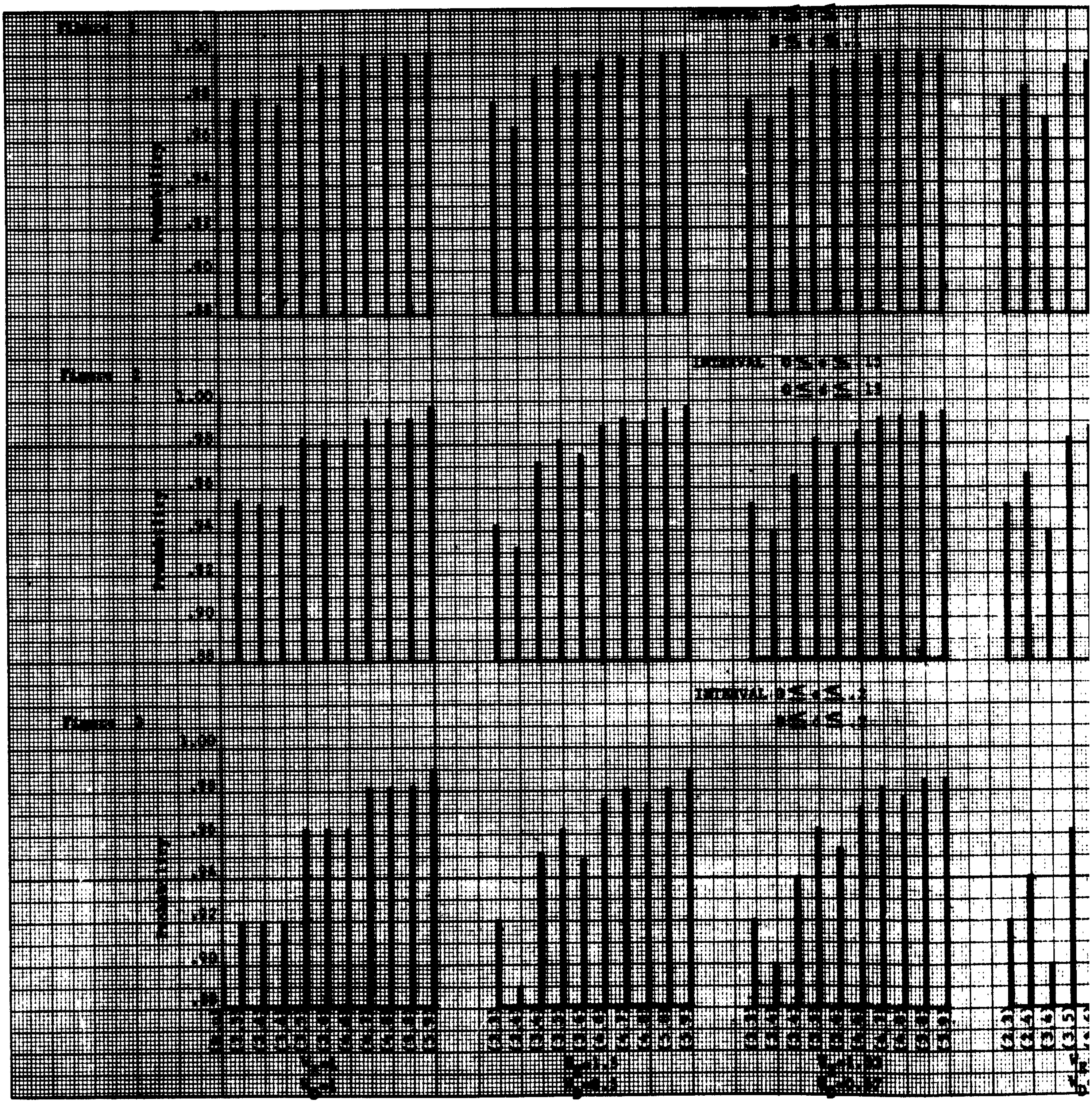
	$0 \le e \le .1$, $0 \le d \le .1$	$0 \le e \le .15$, $0 \le d \le .15$	$0 \le e \le .2$, $0 \le d \le .2$	$0 \le e \le .1$, $0 \le d \le .2$	$0 \le e \le .2$, $0 \le d \le .1$
$W_E=0.67$ $W_D=1.33$					
1s1	.869	.806	.661	.862	.870
1p1-1	.930	.893	.719	.870	.862
1p1p1-2	.977	.952	.920	.939	.958
1p1p1p1-2	.983	.965	.941	.974	.950
1p1p1p1-3	.970	.939	.899	.904	.965
1p1p1p1p1-3	.994	.982	.962	.973	.983
$W_E=0.5$ $W_D=1.5$					
1s1	.854	.783	.642	.848	.901
1p1-1	.946	.916	.745	.901	.848
1p1p1-2	.977	.952	.920	.934	.962
1p1p1p1-2	.987	.972	.951	.976	.962
1p1p1p1-3	.966	.932	.889	.892	.963
1p1p1p1p1-3	.994	.982	.962	.970	.986

Table 23. Average System Probability of Proper Function
(Average Based on Increments of .05 in Probability)

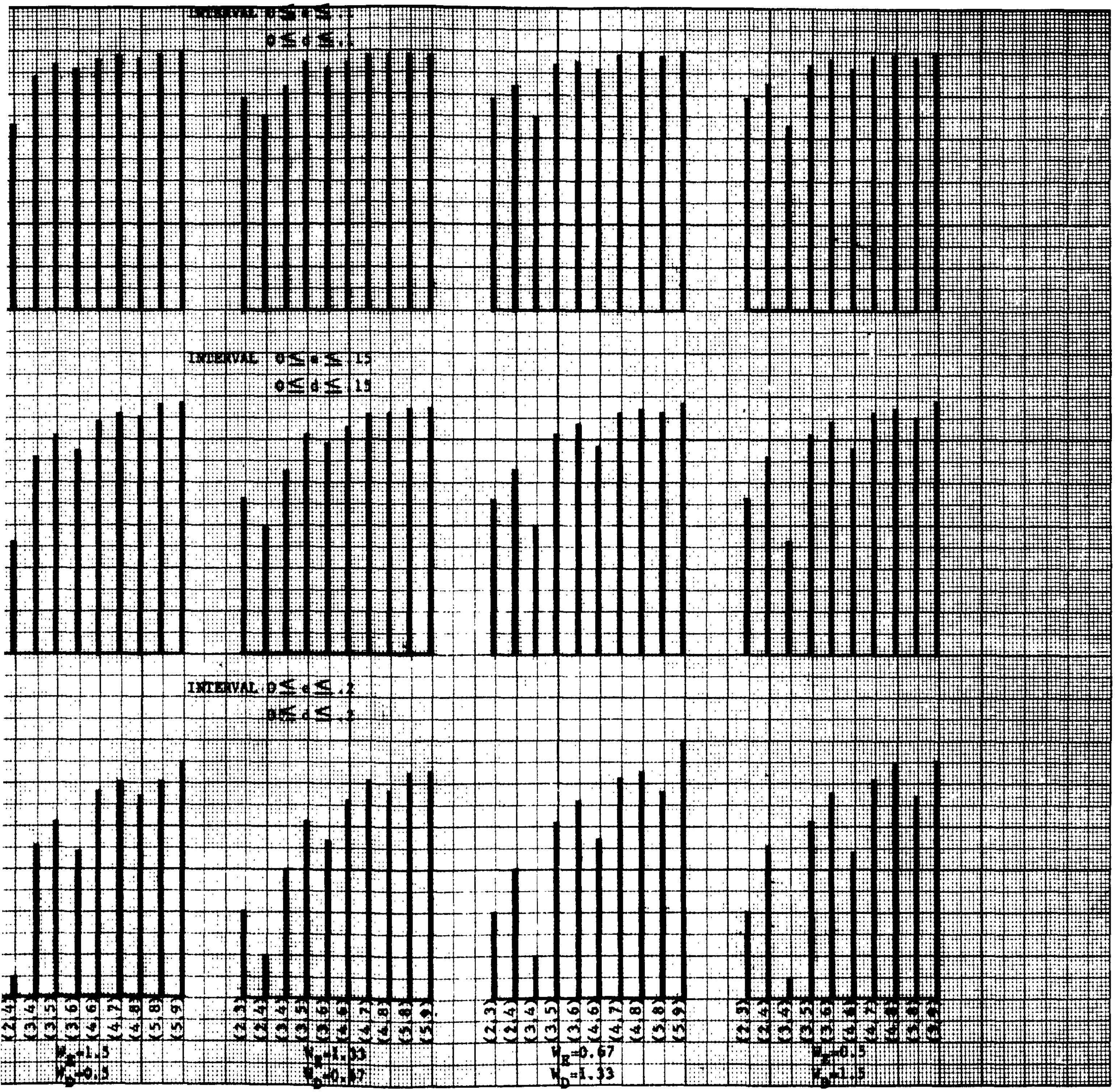
For n Identical Parallel Channels of Which at Least k
Must Function in Order For The System to Function

(k,n)	$0\leq e\leq .1$ $0\leq d\leq .1$	$0\leq e\leq .15$ $0\leq d\leq .15$	$0\leq e\leq .2$ $0\leq d\leq .2$	$0\leq e\leq .1$ $0\leq d\leq .2$	$0\leq e\leq .2$ $0\leq d\leq .1$
$W_E=W_D=1$					
(3,5)	.994	.982	.962	.978	.978
(3,6)	.994	.982	.962	.989	.967
(4,6)	.994	.982	.962	.967	.989
(4,7)	.998	.992	.981	.989	.989
(4,8)	.998	.992	.981	.995	.983
(5,8)	.998	.992	.981	.983	.995
(5,9)	.999	.997	.990	.994	.994
$W_E=1.5$ $W_D=0.5$					
(3,5)	.994	.982	.962	.986	.970
(3,6)	.991	.975	.948	.989	.950
(4,6)	.996	.989	.976	.983	.990
(4,7)	.998	.992	.981	.994	.985
(4,8)	.997	.990	.974	.996	.975
(5,8)	.999	.995	.988	.991	.995
(5,9)	.999	.997	.990	.997	.992
$W_E=1.33$ $W_D=0.67$					
(3,5)	.994	.982	.962	.983	.973
(3,6)	.992	.977	.953	.989	.956
(4,6)	.995	.987	.972	.977	.990
(4,7)	.998	.992	.981	.992	.986
(4,8)	.998	.991	.976	.996	.978
(5,8)	.999	.994	.985	.989	.995
(5,9)	.999	.997	.990	.996	.993
$W_E=0.67$ $W_D=1.33$					
(3,5)	.994	.982	.962	.973	.983
(3,6)	.995	.987	.972	.990	.977
(4,6)	.992	.977	.953	.956	.989
(4,7)	.998	.992	.981	.986	.992
(4,8)	.999	.994	.985	.995	.989
(5,8)	.998	.991	.976	.978	.996
(5,9)	.999	.997	.990	.993	.996
$W_E=0.5$ $W_D=1.5$					
(3,5)	.994	.982	.962	.970	.986
(3,6)	.996	.989	.976	.990	.983
(4,6)	.991	.975	.948	.950	.989
(4,7)	.998	.992	.981	.985	.994
(4,8)	.999	.995	.988	.995	.991
(5,8)	.997	.990	.974	.975	.996
(5,9)	.999	.997	.990	.992	.997

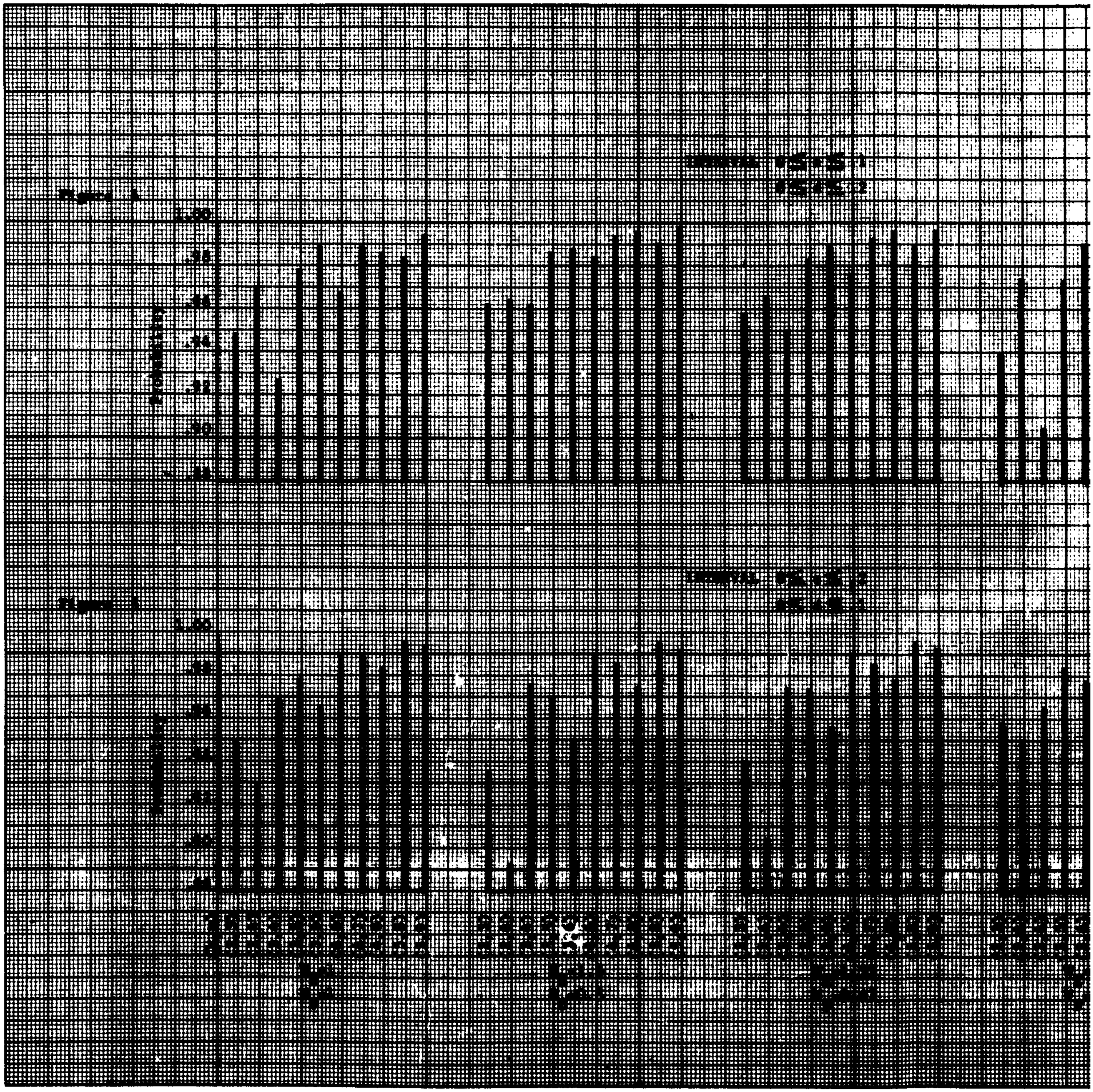
AVERAGE SYSTEM PROPER FUNCTION PROBABILITIES



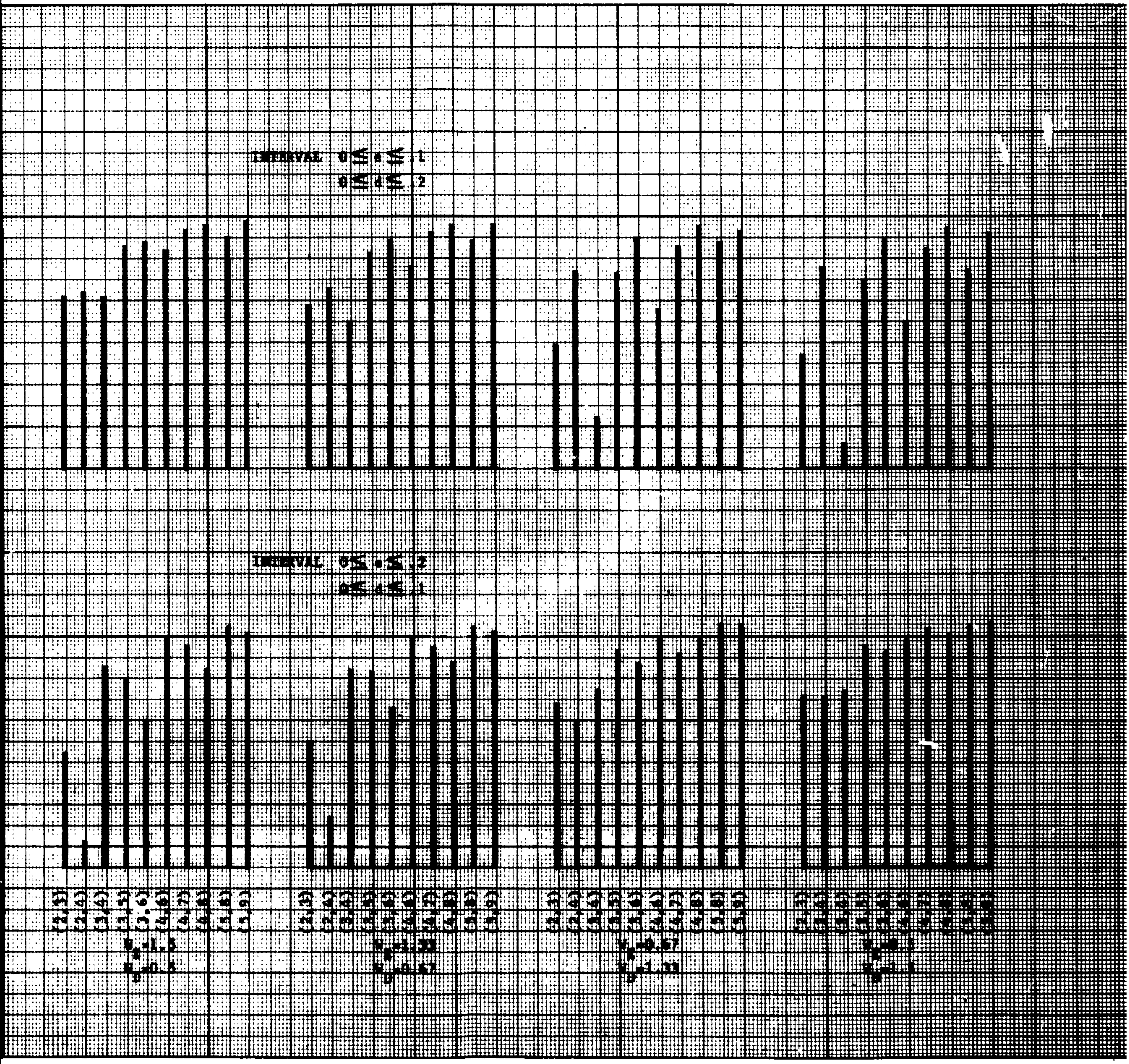
AVERAGE SYSTEM PROPER FUNCTION PROBABILITIES



AVERAGE SYSTEM PROPER FUNCTION PROBABILITIES



AVERAGE SYSTEM PROPER FUNCTION PROBABILITIES



REFERENCES

(1) Kohler, H. W., "Numerical Data on the Probability of Proper Function of Multiple Fuzing Systems", DOFL Report TR-34, January 27, 1954.

(2) Breiter, M. C., "Multiple Fuzing to Increase Proper Function", NBS Report 13/4-95R, November 27, 1950.

(3) Page, C. H. and Karr, P. R., "Multiple Fuzing", NBS Report 13/0-EP-5R, July 28, 1950.

(4) Domsitz, M. G., "An Analysis of Simple Multiple Fuzing Systems," NBS Report 13-0-76M, October 7, 1952.

(5) Dupuy, C. J. and Williamson, E. A., "Multichannel Fuze Systems Feasibility Study", NAVWEPS Report 7179, NOLC, May 1961.

(6) Rau, J. G., "Trichotomous Systems and Their Reliability," NAVWEPS Report 7234, NOLC, probable publishing date, April 1963.

(7) Rau, J. G., "Reliability Determination for General Trichotomous Systems", NOLC Tech Memo 52-68, November 1962.

DISTRIBUTION LIST

	Copies
Chief, Bureau of Naval Weapons Navy Department Washington 25, D. C. Attn: RM	2
RMGA	1
RMMO	3
RRRE	1
Chief of Naval Operations Navy Department Washington 25, D. C. Attn: Code Op03EG	1
Commanding Officer Naval Air Development Center Johnsville, Pa.	1
Commander Naval Missile Center Point Mugu, California	1
Commander Naval Ordnance Test Station China Lake, California Attn: Technical Library	1
Commander Naval Weapons Laboratory Dahlgren, Virginia	1
Commanding Officer Diamond Ordnance Fuze Laboratory Washington 25, D. C.	1
Commander Naval Ordnance Laboratory White Oak Silver Spring, Maryland	1
ITT Federal Laboratories Div. of Inter Telephone and Telegraph Corporation Fort Wayne, Indiana	1
Melpar, Inc. 3000 Arlington Boulevard Falls Church, Virginia	1
Commanding Officer Picatinny Arsenal Dover, New Jersey Attn: FREL	1
Ministry of Aviation, G.W.(G&C) Castlewood House 77/91, New Oxford Street London, W. C. I. Attn: Mr. P.A. Jewell, T.W.P. D-4 Via BUWEPS Code RMMO-2	1
Armed Services Technical Information Agency Arlington Hall Station Arlington 12, Va. Attn: TIPCA	10
Applied Physics Laboratory The Johns Hopkins University 8621 Georgia Avenue Silver Spring, Md.	1
American Optical Company Research Department 14 Mechanic Street Southbridge, Mass.	1
Armour Research Foundation Illinois Inst. of Technology Center Chicago 16, Ill.	1
Bendix Corporation Bendix Systems Division Ann Arbor, Mich.	1
Emertron, Incorporated 1140 East West Highway Silver Spring, Md.	1
INSMAT, Buffalo Engineering Staff 121 Lincoln Ave. Rochester 11, N. Y.	1
Philco Corporation 4700 Wissahickon Avenue Philadelphia 44, Pa.	1

Raytheon Company Missile Systems Division Bedford, Mass.	1
Aerospace Corporation San Bernardino Operations San Bernardino, California Attn: A. D. Morse Bldg. 537, Rm. 922	1
NOLC Washington Office Naval Ordnance Laboratory White Oak, Md.	1
NOLC	
B. F. Husten, Code 50	1
R. A. Boot, Code 50	1
L. L. Parker, Code 50	1
H. A. Bulgerin, Code 51	1
V. A. Brown, Code 52	1
R. L. Bauer, Code 53	1
J. J. Nastronero, Code 54	1
C. R. Hamilton, Code 55	1
H. B. Bryant, Code 56	1
J. J. Pierce, Code 65	1
F. C. Alpers, Code 71	1
W. F. Meggers, Code 72	1
R. V. Talbot, Code 74	1
Technical Library, Code 234	2
Code 522	2